The pathewaie to knowledge, containyng the first principles of Geometrie, as thei maie moste aptly bee applied vnto practise, bothe for vse of instrumentes Geometricall, and Astronomicall: and also for proiection of plattes in euery kinde, and therefore muche necessarie for all sortes of menne.

Geometries verdicte.

All freshe fine wittes by me are filed,
All grosse dull wittes wishe me exiled:
Though no mannes witte reiect will I,
Yet as thei bee, I will them trie.

1574

THE ARGVMENTES of the fower bookes.

The first booke declareth the definitions of the termes and names vsed in Geometry, with certaine of the chiefe groundes whereon the arte is founded. And then teacheth those conclusions, whiche maie serue diuersely in all woorkes Geometricall.

The seconde booke dooeth sette forthe the Thereomes (whiche may be called approued truthes) seruing for the due knowledge and sure proofe of all conclusions and workes in Geometrye.

The third booke intreateth of diuers formes, and sondry protractions thereto belonging, with the vse of certaine conclusions.

The fourth booke teacheth the right order of measuringe all platte formes, and bodies also, by reson Geometricall.

To the gentle Reader.

EXCVSE ME, GENtle Reader if ought bee amiſſe, ſtraunge pathes are not trodẽ al truly at the firſt: the way muſte needes be comberous, wher none hathe gone before. VV here no man hath geuen lighte, lighte is it to offend, but when the light is ſhewed once, light is it to amende. If my light may ſo light ſome other, to eſpie and marke my faultes, I wiſh it may ſo lighten them, that thei maie voide offence. Of ſtaggeryng and ſtomblyng, and vnconſtaunte turmoylyng: often offendyng, and ſeldome amendyng, ſuche vices to eſchewe, and their fine wittes to ſhewe; that thei maie winne the praiſe, and I to holde the candle, whileſt thei their glorious woorkes with eloquence ſette foorthe, ſo cunnyngly inuented, ſo finely indited, that my bookes maie ſeme worthi? to occupie no roume. For neithe is my witte ſo finely filed, neither my learnyng ſo largely lettered, neither yet my laiſure ſo quiet and vncombered, that I maie performe iuſtely ſo learned a labour, or accordyngly to accompliſhe ſo haultie an enforcemente,

yet may I thinke thus: This candle did I light: this light haue I kindeled: that learned menne maie see, to practise their pennes, their eloquence to aduaunce, to regi-ster their names in the booke of memorie, I drew the platte rudelie, whereon they maye builde, whom God hath indued with learnyng and liuelihod. For liuyng by laboure doth learning so hinder, that learning serueth liuynge, whiche is a peruers trade. Yet as carefull familie shall cease hir cruell calling, and suffre anie laiser to learnynge to repaire, I will not cease from trauaile the pathe so to trade, that finer wittes maie fashion them selues with suche glimsing dull light, a more complete woorke at laiser to finishe, with inuencion agreable, and aptnes of eloquence.

And this gentle Reader I hartelie protest, where erroure hath happened I wishe it redreste.

TO THE MOST NOBLE AND PVISSAVNT PRINCE EDVVARD THE SIXTE BY THE

grace of God, of Englande, Fraunce, and Irelande Kyng, defendour of the faithe, and of the churche of Englande and Irelande in earth the supreme head.

IT IS NOT VNKNOVVEN to your maiestie, moste soueraigne Lorde, what greate disceptation hath been amongest the wyttie men of all nacions, for the exacte knoweledge of true felicitie, bothe what it is, and wherein it consisteth: touchynge whiche thyng, their opinions almoste were as many in numbre, as were the persones of them, that either disputed or wrote thereof. But and if the diuersitie of opinions in the vulgar sorte, for placyng of their felicitie shall bee considered also, the varietie shall be founde so great, and the opinions so dissonante, yea plainly monsterous, that no honest witte would vouchesafe to lose tyme in hearyng them, or rather (as I maye saie) no witte is of so exacte remembrãce, that can consider together the monstrous multitude of theim all. And yet not withstandyng this repugnaunt diuersitie, in twoo thinges do thei all agree. First all doe agre, that felicitie is and ought to bee the stop and ende of all their doynges, so that he that hath a full desire to any thyng, how so euer it be estemed of other men, yet he estemeth hym self happie, if he maie obtain it: and contrary waies vnhappie if he can not attaine it. And therefore doe all men putte their whole studie to gette that thyng, wherin thei haue perswaded theim self that felicitie doeth consiste. Wherefore some

whiche put their felicitie in feedyng their beastes, thinke no pain to be harde, nor no deede to be unhoneste, that maie be a meanes to fill that foule panche. Either whiche put their felicitie in plaie and idle pastymes, iudge no tyme euill spent, that is emploied thereaboute: nor no fraude unlawfull that maie farther their winnyng. If I should particularly ouercunne but the common sortes of men, whiche put their felicitie in their desires, it would make a greate booke of it self. Therefore will I let them all go, and conclude as I began, That all men employ their whole erdeuour to that thing, wherein they thinke felicitie to stand, whiche thyng who so listeth to marke exactly, shall be able to espie and iudge the natures of all men, whose conuersacion he doth knowe, though they use great dissimulacion to colour their desires, especially when thei perceiue other mennes to mislike that, whiche thei so muche desire: for no man would gladly haue his appetite improued. And hereof commeth that wonderthing wherein all agree, that euery man would moste gladly win all other men to his secte, and to make theim of his opinion, and as farre as he dare, will dispraise all other mennes iudgementes, and praise his owne waies onely, onles it bee when he dissimuleth, and that for the fartheraunce of his owne purpose. And this propertie also doeth giue great light to the full knowledge of mennes natures, whiche as al men ought to obserue, so Princes aboue other haue mooste cause to marke for sundrie occasions, whiche maie lye theim on, whereof I shall not nede to speake any farther, consideryng not onely the greatenesse of witte, and exactnesse of iudgemente, whiche God hath lente unto your highnes persone, but also the moste graue wisedom, and profounde knowledge of your Maiesties moste honorable counsaile, by whom your highnes maie so sufficiently understande all thynges conuenient, that lesse shall it nede to understande by priuate readyng, but yet not utterly to refuse to reade as often as occasion maie serue, for bookes dare speake, when menne feare to displease. But to returne agayne to my firste matter, if

none

none other good thing maie be lerned at their maners, whiche so wrongfully place their felicitie, in so miserable a condition (that while they thinke them selfes happy, their felicitie must nedes seme unluckie, to be by them so euill placed) yet this may men learne at them, by those two spectacles to espye the secrete natures and dispositions of others, whiche thyng unto a wise man is muche auailable. And thus will I omit this great rablement of unhappie hap, and wil come to thre other sortes of a better degre, whereof the one putteth felicitie to consist in power and royaltie. The second sorte unto power annexeth worldly wisedome, thinkyng him full happie, that could attain those two, wherby he might not onely haue knowledge in all thynges, but also power to bring his desires to ende. The thyrd sorte estemeth true felicitie to consist in wisedome annexed with vertuouse maners, thinkyng that they can take harme of nothing, if they can with their wisedome ouercome all vyces. Of the firste of those thre sortes there hath been a great numbre in all ages, yea many mightie kinges and great gouerneures, whiche cared not greately howe they myght atchieue their pourpose, so that they dyd preuayle: Nor did not take any greater care for gouernaunce, then to kepe the people in onely feare of them. Whose common sentence was alwaies this: Oderint dum metuant, And what good successe suche menne had, all histories doe report. Yet haue they not wanted excuses: yea Iulius Cæsar (whiche in dede was of the seconde sorte) maketh a kinde of excuse by his common sentence, for theim of that firste sorte, for he was euer wonte to saie:

εἴπερ γὰρ ἀδικεῖν χρὴ, τυραννίδος περικάλλιστον ἀδικεῖν. τ' ἄλλα δ' εὐσεβεῖν χρεῶν.

Whiche sentence, I wishe had neuer been learned out of Grecia. But nowe to speake of the seconde sorte, of whiche there hath been verie many also, yet for this presente time amongest them all, I wyll take the exaumple of kynge Philippe of Macedonie, and of Alexander his sonne,

that

that valiaunt conquerour. First of kinge Phylippe it appeareth by his letter sente vnto Aristotle that famous philosopher, that he more delited in the birthe of his sonne, for the hope of learning and good education, that might happen to hym by the said Aristotle, then he didde reioyse in the continuaunce of his succession, for these were his wordes and his whole epistle, worthye to bee remembred and registred euery where.

Φίλιππος Ἀριστοτέλει χαίρειν.

Ἴσθι μοι γεγονότα υἱόν. πολλὴν οὖν τοῖς θεοῖς χάριν ἔχω, οὐχ οὕτως ἐπὶ τῇ γενέσει τοῦ παιδός, ὡς ἐπὶ τῷ κατὰ τὴν σὴν ἡλικίαν αὐτὸν γεγονέναι. ἐλπίζω γὰρ αὐτὸν ὑπὸ σοῦ τραφέντα καὶ παιδευθέντα ἄξιον ἔσεσθαι καὶ ἡμῶν καὶ τῆς τῶν πραγμάτων διαδοχῆς.

That is thus in sense.

Philip vnto Aristotle sendeth gretyng.

You shall vnderstande, that I haue a sonne borne, for whiche cause I yelde vnto God moste hartie thankes, not so muche for the byrthe of the childe, as that it was his chaunce to be borne in your tyme. For my trust is, that he shall be so brought vp, and instructed by you, that he shall become worthie not only to be named our sonne, but also to be the successour of our affaires.

And his good desire was not all vayne, for it appered that Alexander was neuer so busied with warres (yet was he neuer out of moste terrible battaile) but that in the middes thereof he had in remembraunce his studies, and caused in all countreies as he wente, all straunge beastes,

fowles

fowles and fishes, to be taken and kept for the ayde of that knowledge, whiche he learned of Aristotle: And also he had with him alwayes a great numbre of learned men. And in the most busye tyme of all his warres against Darius kyng of Persia, when he harde that Aristotle had putte forthe certaine bookes of suche knowledge wherein he hadde before studied, hee was offended with Aristotle, and wrote to hym this letter.

Ἀλέξανδρος Ἀριστοτέλει εὖ πράττειν.

Οὐκ ὀρθῶς ἐποίησας ἐκδοὺς τοὺς ἀκροαματικοὺς τῶν λόγων, τίνι γὰρ διοίσομεν ἡμεῖς τῶν ἄλλων, εἰ καθ' οὓς ἐπαιδεύθημεν λόγους, οὗτοι πάντων ἔσονται κοινοί, ἐγὼ δὲ βουλοίμην ἂν ταῖς περὶ τὰ ἄριστα ἐμπειρίαις, ἢ ταῖς δυνάμεσι διαφέρειν. ἔρρωσο. that is

Alexander vnto Aristotle sendeth greetyng.

You haue not doone well, to put forthe those bookes of secrete phylosophy intituled, ἀκροαματικοί. For wherein shall we excell other, yf that knowledge that wee haue studied, shall be made commen to all other men, namely sithe our desire is to excelle other men in experience and knowledge, rather then in power and strength. Farewell.

By whiche lettre it appeareth that hee esteemed learninge and knowledge aboue power of men. And the like iudgement did he vtter, when he beheld the state of Diogenes Cinicus, adiudginge it the beste state next to his owne, so that he said: If I were not Alexander, I wolde wishe to be Diogenes. Whereby appeareth, how he esteemed learning, and what felicity he putte therin, reputing all the worlde saue him selfe to bee inferiour to Diogenes And by all coniectures, Alexander did esteeme Diogenes one of them whiche contemned the vaine estimacion of the

disceitfull worlde, and put his whole felicitie in knowledge of vertue, and practise of the same, though some reporte, that he knewe more vertue then he folowed: But what so euer he was, it appeareth that Socrates and Plato and many other did forsake their liuynges and sell awaye their patrimonie, to the intent to seeke and trauaile for learnyng whiche examples I shall not neede to repeate to your Maiestie, partly for that your highnes doeth often reade them and other like, and partely sith your maiestie hath at hande suche learned Schoolemaisters, whiche can muche better then I, declare theim vnto your highnes, and that more largely also then the shortenesse of this Epistle will permitte. But this maie I yet adde, that Kyng Salomon whose renoume spred so farre abroade, was very greatly estemed for his wonderfull power and exceadyng treasure, but yet much more was he estemed for his wisedome. And hym self doeth bear witnes, that wisedome is better then precious stones, yea all thynges that can be desired are not to be compared to it. But what needeth to alledge one sentence of hym, whose bookes altogither do none other thing, then set forthe the praise of wisedome and knowledge? And his father Kinge Dauid ioineth vertuous conuersacion and knowledge together, as the summe of perfection and chief felicitie. Wherefore I maie iustely conclude, that true felicitie doeth consiste in wisedome and vertue. Then if wisedome bee as Cicero defineth it, Diuinarum atque humanarum rerum scientia, then ought all menne to trauaile for knowledge in matters both of religion and humaine doctrine, if he shall be counted wise, and able to attain true felicitie: but as the studie of religious matters is mooste principall, so I leaue it for this tyme to them that better can write of it then I can. And for humaine knowledge thys will I boldly say, that who soeuer will attain true iudgement therin, must not onely trauail in the knowledge of the tonges, but muste also before all other artes, taste of the Mathematicall sciences, specially Arithmetike and Geometrie, without whiche it is not possible to

attayne full knowledge in any arte. Whiche may suffi-ciently by gathered by Aristotle not onely in his bookes of demonstration (which can not be vnderstand without Geometrie) but also in all his other woorkes. And before hym Plato his maister wrote this sentence on his schole house dore. Αγεωμέτρητος ουδεὶς εισίτω. Let no man entre here (saith he) without knowledge in Geometrie. Wherfore moste mightie prince, as your moste excellent Maiestie appeareth to bee borne vnto mooste perfecte felicitie, not onely by reason that GOD moued with the longe praiers of this realme, did send your highnes as a most comfortable inheritour to the same, but also in that your Maiestie was borne in the time of suche skilfull schoolemaisters and learned teachers, as your highnes doth not a little reioyse in, and profite by theim in all kind of vertue and knowledge. Emongst whiche is that heauenly knowledge most worthely to be praised, whereby the blindnes of errour and superstition is exiled, and good hope conceiued that al the sædes and fruites thereof, with all kindes of vice and iniquite, wherby vertue is hindered, and iustice defaced, shall bee cleane extirped and rooted out of this realme, whiche hope shall increase more and more, if it may appeare that learning be esteemed and florishe within this realme. And all be it the chief learning be the diuine Scriptures, whiche instructe the minde principally, and next therto the lawes politike, whiche moste specially defende the right of gooddes, yet is it not possible, that those twoo can long be well vsed, if that aide want that gouerneth health and expelleth sicknes, whiche thing is done by Phisicke, and these require the helpe of the seuen liberall sciences, but of none more then of Arithmetike and Geometrie, by whiche not onely greate thynges are wroughte touching accomptes in al kindes, and in suruaiyng and measuring of landes, but also all artes depend partely of them, and building whiche is moste necessary can not be without them whiche thing considering, moued me to helpe to serue your maiestie in this poincte, as well as other waies, and to

do what may be in me, that not only they which studie principally for learning, may haue furderance by my pore help but also those whiche haue no tyme to trauaile for exacter knowledge, may haue some helpe to vnderstande in those Mathematicall artes, in whiche as I haue all readye set forthe sumwhat of Arithmetike, so God willing I intend shortely to set forth a more exacter worke thereof. And in the meane ceason for a taste of Geometrie, I haue sette forthe this small introduction, desiring your grace not so much to beholde the simplenes of the woorke, in comparison to your Maiesties excellencye, as to fauour the edition thereof, for the ayde of your humble subiectes, whiche shall thinke them selues more and more dayly bounden to your highnes, if when they shall perceaue your graces desyre to haue theym profited in all knowledge and vertue. And I for my pore ability considering your Maiesties study for ye increase of learning generally through al your highenes dominions, and namely in the vniuersities of Oxforde and Camebridge, as I haue an earnest good will as far as my simple seruice and small knowledge will suffice, to helpe toward the satisfiyng of your graces desire, so if I shall perceaue that my seruice may be to your maiesties contentation, I will not only put forth the other two bookes, whiche shoulde haue beene sette forth with these two, yf misfortune had not hindered it, but also I will sette forthe other bookes of more exacter arte, bothe in the Latine tongue and also in the Englyshe, whereof parte bee all readye written, and newe instrumentes to theym deuised, and the residue shall bee eanded with all possible speede. I was boldened to dedicate this booke of Geometrie vnto your Maiestye, not so muche bycause it is the firste that euer was sette forthe in Englishe, and therefore for the noueltye a straunge presente, but for that I was perswaded, that suche a wyse prince doothe desire to haue a wise sorte of subiectes. For it is a kynges chiefe reioysinge and glorye, if his subiectes be riche in substaunce, and wytty in knowledge:

and

and contrarye wayes nothing can bee more greuouse to a noble Kyng, then that his Realme should be other beggerly or full of ignoraunce: But as God hath geuen your grace a realme bothe riche in commodities and also full of wyttie men, so I truste by the readyng of wyttie artes (whiche be as the whette stones of witte) they muste needes increase more and more in wisedome, and peraduenture fynde some thynge towarde the ayde of their substaunce, whereby your grace shall haue newe occasion to reioyce, seyng your subiectes to increase in substaunce or wisedome, or in both. And they again shall haue new and new causes to pray for your Maiestie, perceiuyng so graciouse a minde towarde their benefite. And I truste (as I desire) that a great numbre of gentlemen, especially about the courte, whiche vnderstande not the Latine tong, or els for the hardnesse of the matter could not away with other mens writtyng, will fall in trade with this easie forme of teachyng in their vulgar tong, and so employe some of their tyme in honeste studie, whiche were wont to bestowe moste part of their tyme in triflyng pastime: for vndoubtedly if thei mean either your maiesties seruice, other their owne wisedome, they will be content to employ some tyme aboute this honeste and wittie exercise. For whose encouragement to the intent they maye perceiue what shall be the vse of this science, I haue not onely written somewhat of the vse of Geometrie, but also I haue annexed to this booke the names and brefe argumentes of those other bookes whiche I will sette forthe hereafter, and that as shortely as it shall appeare vnto your Maiestie by coniecture of their diligent vsyng of this first booke, that they wyll vse well the other bookes also. In the meane season, and at all times I will be a continuall petitioner, that God may worke in all Englishe hartes an ernest mynde to all honest exercises, whereby they may serue the better your Maiestie and the Realme. And for your highnes I besech the moste mercifull God, as he hath most fauourably sent you vnto vs, as our chiefe comforter in

earthe, so that he will increase your Maiestie daiely in all vertue and honor with moste prosperouse successe, and augment in vs your moste humble subiectes, true loue to godward, and iust obedience toward your highnes with all reuerence and subiection.

At London the xxviij. daie of Iannuaie. M. D. L I.

Your Maiesties moste humble seruant and obedient subiect, Robert Recorde.

THE DEFINICIONS of the principles of GEOMTRIE.

GEOMETRIE teacheth the drawyng, Meaſuryng, and proporcion of figures, but in as muche as no figure can bee drawen, but it muſte haue certaine boundes and incloſures of lines: and euery line alſo is begon and ended at ſome certaine pricke, firſte it ſhall bee meete to knowe theſe ſmaller partes of euerie figure, that thereby the whole figures maie the better be iudged, and diſtincte in ſonder.

A Point or a Pricke, is named of the Geometricians that ſmall and vnſenſible ſhape, whiche hath in it no partes, that is to ſaie: neither length, breadth, nor depth. But as this exactnes of definitiō, is more meeter for onely Theorike ſpeculation, then for practiſe, and outwarde worke (conſideryng that myne intente is to applie all theſe whole principles to worke) I thinke meeter for this purpoſe, to call a pointe or pricke, that ſmall print of penne, pencile, or other inſtrumēt, whiche is not moued, nor drawen from his firſte touche, and therefore hath no notable length nor breadth: as this example doeth declare. ∴ • A poincte.

Where I haue ſet .iij. prickes, eche of them hauyng bothe length and breadth, though it be but ſmall, and therefore not notable.

Now of a greate nomber of theſe prickes, is made a line, as you maie perceiue by this forme enſuyng. ....

Where as I haue ſet a nōber of prickes, ſo if you with your penne, will ſet in more other prickes betwene euery twoo of theſe, then will it be a line, as here you maie ſe ———————— A line.
and this line, is called of Geometricians, lēgth without bredth.

But as thei in their Theorikes (whiche are onely mynde

woorkes) dooe precisely vnderstande these definitions, so it shalbe sufficiente for those men, whiche seeke the vse of the same thinges, as sense maie duely iudge them, and applie to handie woorkes, if thei vnderstande them so to bee true, that outward sense can finde none errour therein.

Of lines there be twoo principall kindes. The one is called a right, or straight line, and the other a croked line.

A streight line.

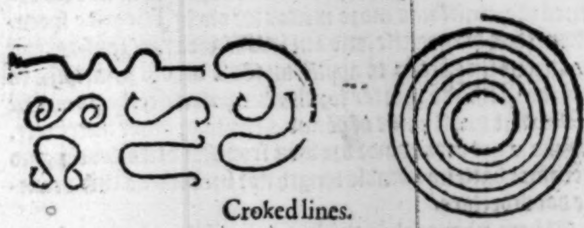
A straight line, is the shortest that maie bee drawen betwene twoo prickes.

A croked line

And all other lines, that goe not right forthe from pricke to pricke, but boweth any waie, suche are called croked lines as in these examples folowyng, ye maie se, where I haue set but one forme of a straight line, for more formes there bee not, but of croked lines there bee innumerable diuersities, whereof for examples, some I haue sette here.

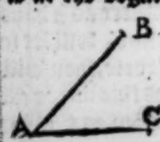
A right line.

Croked. lines.



Croked lines.

So now you must vnderstande, that euery line drawen betwene twoo prickes, whereof the one is at the beginnyng, and the other at the ende.



Therefore, when so euer you doe see any formes of lines, to touche at one notable pricke, as in this example, then shall

you not call it one crooked line, but rather twoo lines: in as muche as there is a notable and sensible angle by A. whiche euermore is made the meetyng of twoo seuerall lines. And likewaies shall you iudge of this figure, whiche is made of twoo lines, and not of one onely. An Angle.

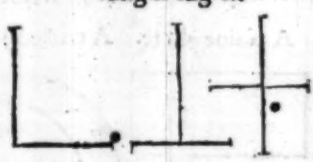
So that when so euer any suche meetyng of lines doeth happē, the place of their metyng is called an angle or corner.

Of angles there bee three generall kindes: a sharpe angle, a square angle, and a blunte angle. The square angle, whiche is commonly named a right corner, is made of twoo lines metyng together in forme of a squire, whiche twoo lines if thei bee drawen forthe in length, will crosse one an other: as in the examples folowyng you maie see. A right angle

A sharpe angle is so called, because it is lesser then is a square angle, and the lines that make it, dooe not open so wide in their departyng, as in a square corner, and if thei bee drawen crosse, all fower corners will not bee equall. A sharpe corner.

A blunte or brode corner, is greater then is a square angle, and his line doe parte more in sonder, then in a right angle, of whiche all take these examples. A bluntangle

Right angles.



And these angles (as you se) are made partly of streight lines, partly of crooked lines, and partely of bothe together. Howbeit in right

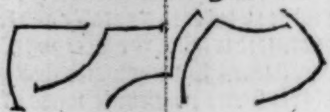
Sharpe angles.

angles I haue put none example of crooked lines, because it

 would

would trouble a lerner to iudge them: for their true iudgemente doeth appertaine to arte perspectiue, and as I maie saie, rather to reason then to sense.

Blunt or brode angles.



But nowe as of many prickes there is made one line, so of diuerse lines are there made sundrie fourmes, figures, and shapes, whiche all yet be called by one proper name, Platte fourmes, and thei haue bothe length and breadth, but yet no depenesse.

A plat forme.

And the boundes of euery platte forme are lines; as by the examples you maie perceiue.

Of platte formes some bee plaine, and some bee crooked, and some partly plaine, and partly crooked.

A plain plat. A plaine plat is that, whiche is made all equall in height, so that the middle partes, neither bulke vp, neither shrinke doune more then the bothe endes.

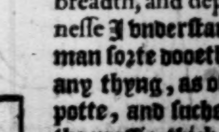
For when the one parte is higher then the other, then is it named a croked platte.

A croked plat

And if it be partly plaine, & partly croked, then is it called a Mixte platte, of all whiche, these are examples.

A plaine platte.

A croked platte.



A mixte platte.

And as of many prickes is made a line, and of diuerse lines one Platte forme, so of many plattes is made a bodie, whiche conteineth Lengthe, breadth, and depenesse. By Deepenesse I vnderstande, not as the comman sorte dooeth, the hollownesse of any thyng, as of a welle, a diche, a potte, and suche like, but I meane the massie thickenesse of any bodie, as

A bodie.

Depenesse.

as in example of a potte: the deepenesse is after the common name, the space from his brimme to his bottome. But as I take it here, the deepenesse of his bodie, is his thickenesse in the sides, whiche is an other thyng cleane different from the deepenesse of his holownesse, that the common people meaneth.

Nowe all bodies haue platte formes for their boundes, so in a Die (whiche is called a cubicke bodie) by Geometricians and an ashler of Masons, there are sixe sides, whiche are sixe platte formes, and are the formes of the Die. Cubike. Asheler.

But a Globe, (whiche is a bodie rounde as a boule) there is but one platte fourme, and one bounde, and these are the examples of them bothe. A globe.

A dye or ashler, A Globe.

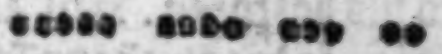
But bicause you shall not muse what I doe cal a bound, I meane thereby a generalle name, betokening the beginnyng, ende, and side, of any forme. A bounde.

A forme, figure, or shape, is that thyng that is inclosed within one bonde, or many bondes, so that you vnderstande the shape, that the eye doeth discerne, and not the substaunce of the bodie. Forme figure.

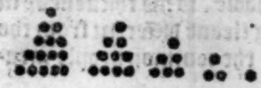
Of figures there bee many sortes, for either thei be made of prickes, lines, or plat formes. Notwithstanding to speake properly, a figure is made by platte formes, and not of bare lines vnclosed, neither yet of prickes.

Yet for the lighter forme of teachyng, it shall not bee vnsemely to call all suche shapes, formes and figures, whiche the eye maie discerne distinctly.

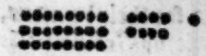
And first to begin with prickes, there maie bee made diuerse formes of them, as partly heredoeth folowe.



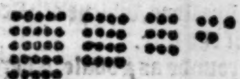
A linearie number.



Trianguler numbers.

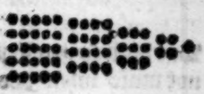


Long square numbers.



Iuste square numbers.

A three cornered spire.



A square spire.

And so maie there bee infinite formes more, whiche I omitte for this tyme, consideryng that their knowledge apertaineth more to Arithmetike figurall, then to Geometrie.

But yet one name of a pricke, whiche he taketh rather of his place, then of his forme, maie I not ouerpasse. And that is, when a pricke standeth in the middle of a circle (as no circle can bee made by compasse without it) then is it called a centre. And therefore doe Masons, and other woorke men call that patron, a centre, wherby thei drawe the lines, for iuste hewyng of stones for arches, vaultes, and chimneyes, because the chief vse of that patron is wrought, by findyng that pricke or centre, whiche all the lines are drawen, as in the thirde booke it doeth appere.

A centre.

Lines make diuerse figures also, though properly thei maie not bee called figures, as I saied before (vnlesse the li-

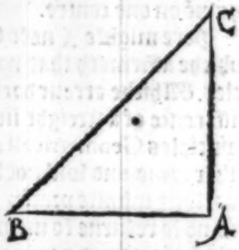
nes doe close) but onely for easie maner of teachyng, all shall be called figures, that the eye di- discerne, of whiche this is one, when one line lieth flat (whiche is named the grounde line) and an other commeth downe on it, and is called a perpendiculare, or plumbe line, as in this example you maie see. Where A. B. is the grounde line, and C. D. the plumbe line.

A grou͂d line.

A perpendiculare line.

A plu͂be line.

And likewaies in this figure there are three lines, the ground line whiche is A. B. the plumbe line, that is A. C. and the bias line, whiche goeth from the one of them to the other, and lieth against the right corner in suche a figure, whiche is here C. B.



But consideryng that I shall haue occasion to declare sundrie figures anon, I will firste shewe some certaine varieties of lines that close no figures, but are bare lines, and of the other lines will I make mention in the description of the figures.

Paralleles, or Gemowe lines bee suche lines as bee drawen forthe still in one distaunce, and are no nerer in one place, then in an other, for an if thei bee nerer at one eande then at the other, then are thei no paralleles, but maie bee called bought lines, and loe here examples of them both.

Torturouse paralleles.

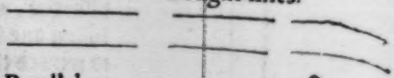
Paralleles. Gemovve lines.

I haue

I haue added also paralleles torturouse, whiche bowe contrarie waies with their two endes: and paralleles circular, whiche bee like vnperfect cōpasses: For if thei bee whole circles, thē are thei called concētriks that is to saie, circles drawē on one centre.

Concentrikes

Paralleles. bought lines.



Paralleles circular,

Concentrikes,

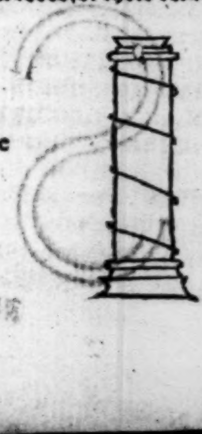
Here mighte I note the errour of good Albert Durer, whiche affirmeth that no perpendicular lines can bée paralleles. Whiche errour doeth spryng partly of ouersight of the difference of a streight line, and partly of mistakyng certain principles Geometricall, whiche al I will let passe vntill an other tyme, and will not blame hym, whiche hath deserued worthily infinite praise.

A tvvine line

And to returne to my matter, an other fashiō line is there, whiche is named a twine or twist line, & it goeth as a wreith about some other bodie. And an other sorte of lines is there, that is called a spirall line, or a worme line, whiche representeth an apparant forme of many circles, where there is not one in deede: of these two kindes of lines, these be examples.

A spirall line. A vvorme line.

A twiste line.

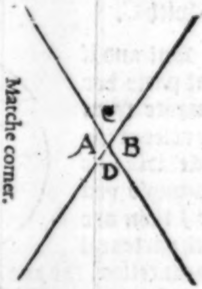


A spirall line.

A touche line, is a line that runneth a long by the edge of a circle, onely touchyng it, but doeth not crosse the circumference of it, as in this example you maie see. A touch line.

And when that a line dooeth crosse the edge of the circle, then is it called a corde, as you shall see anon in the speakyng of circles. A corde.

In the meane season must I not omit to declare, what angles bee called matche corners, that is to saie, suche as stand directly one against the other, when twoo lines bee drawen a crosse, as here appereth. Matche corner.



When A. and B. are matche corners, so are C. and D, but not A. and C. neither D. and A.

Nowe will I beginne to speake of figures, that bee properly so called, of whiche all bee made of diuers lines, excepte onely a circle, an egge forme, and a tunne forme, whiche three haue no angle, and haue but one line for their bounde, and an eye forme, whiche is made of one line, and hath an angle onely.

A circle is a figure made and enclosed with one line, and hath in the middle of it a pricke or centre, from whiche al the lines that bee drawen to the circumference are equall all in length, as here you see. A circle.

And the line that encloseth the whole compasse, is called the circumference. Circūference.

And all the lines that bee drawen crosse the circle, and goe by the centre, are named Diameters, whose halfe, I meane from the centre to the circum- A diametre.

Semidiameter

ference any waie, and is called the semidiameter, or halfe diameter.

A corde or a stryng line.

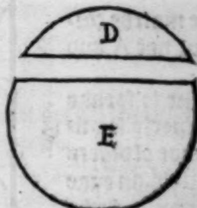
But and if the line goe crosse circle, and passe beside the centre, then is it called a Corde, or a Stryng line, as I saied before, and as this example sheweth: where A. is the corde.

An arche line

A bovve line

And the compassed line that answereth to it, is called and Arche line, or a Bowe line, whiche here is marked with B. and the Diameter with C.

But and if that parte bee separate from the reste of the circle (as in this exaumple you see) then are both partes called cantelles, the one the greater cantell, as E. and the other the lesser cantell, as D. And if it bee parted iuste by the centre (as you see in F.) then is it called a semicircle, or halfe compasse.



A cantle.

A semie circle

Sometymes it happeneth that a cantell is cutte out with twoo lines, drawen from the centre to the circumference (as G, is) and then make it bee called a Nooke cantell, and if it be not parted from the reste of the circle (as you see in H.) then is it called a nooke plainlie, without any addition. And the compassed line in it, is called an Arche line, as the example here doeth shewe.

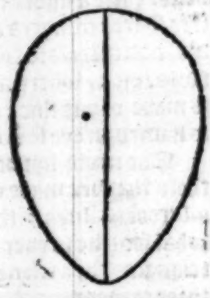
A nooke cantle.

A nooke.

An

An arche,

Nowe haue you heard as touchyng circles, meetely sufficient instruction, so that it should seme nœdelesse to speake any more of figures in that kinde, saue that there doeth yet remaine twoo fourmes of an imperfecte circle, for it is like a circle that were brused, and thereby did runne out eande longe one waie, whiche fourme Geometricians dooe call an Egge fourme, because it doeth represente the figure and shape of an Egge duely proportioned (as this figure sheweth) hauyng the one ende greater then the other. An egge fourme.



An Egge forme.

A tunne fourme.

For if it bee like the figure of a circle pressed in lengthe, and bothe sides like bigge, then is it called a tunne fourme, or barrell fourme, the right makyng of whiche figures, I will declare hereafter in the thirde booke. A tunne or barrell fourme

An other fourme there is, whiche you maie call a Nutte fourme, and is made of one line, muche like an egge fourme saue that it hath a sharpe angle.

And it chaunceth sometyme that there is a right line drawen crosse these figures, and that is called an axeline, or axtree. Howbeeit, properly that line that is called an axetree, whiche goeth through the middle of a Globe, for as a Diameter is in a circle, so is an axe line or axetree in a Globe, that line that goeth from side to side, and passeth in the An axetree or axe line.

middle of it. And the twoo poinctes that suche a line maketh in the vtter bounde or platte of the Globe, are named Polis, whiche you maie call aptly in Englishe, tourne poinctes: of whiche I doe more largely intreate, in the booke that I haue written of the vse of the Globe.

But to retourne to the diuersities of figures that remain vndeclared, the moste simple of them are suche ones, as bee made but of twoo lines, as are the cantle of a circle, and the halfe circle, of whiche I haue spoken alreadie. Likewise the halfe of an egge fourme, the the cantle of an egge fourme, the halfe of a tunne fourme, and the cantle of a tunne fourme, and beside these a figure muche like to a tunne fourme, saue that it is sharpe cornered at both the endes, and therefore dooeth consiste of twoo lines, where a tunne fourme is made of one line, and that figure is named an eye fourme.

An eye forme

The nexte kynde of figures are those that bee made of three lines, either be all right lines, all crooked lines, either some right, and some crooked. But what fourme so euer thei bee of, thei are named generally triangles, for a triangle is nothyng els to saie, but a figure of three corners.

A triangle.

And this is a generall rule, loke how many lines any figure hath, so many corners it hath also, if it be a plat forme and not a body. For a bodie hath diuers lines metyng sometyme in one corner.

Now to giue you example of triangles, there is none whiche is all of crooked lines, and maie bee taken for a portion of a Globe, as the figure marked with A.

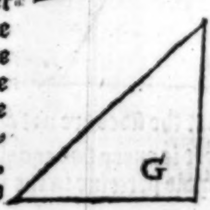
An other hath twoo compassed lines and one right line, and is as the portion of halfe a Globe, example of B.

An other hath but one compassed line, and is the quarter of a circle, named a quadrate, and the right lines make a right corner, as you sein C. Other lesse then it as you see D, whose righte lines make a sharpe corner, or greater then a quadrate, as is F, and then the right lines of it doe make a blunte corner.

Also some triãgles haue all right lines, and thei bee distincted in sonder by their angles, or corners, for either their corners be all sharpe, as you see in the figure E. Other twoo sharpe and one right square, as in the figure G. other twoo sharp and one blunt, as in the figure H.

There is also an other distinction of the names of triangles, accordyng to their sides, whiche either be all equall, as in the figure E, and that the Greekes do call Isopleuron, and Latine menne ισόπλευρον. æquilaterum: and in Englishe it maie bee called a threlike triangle. either els twoo sides bee equall, and the third vnequall, whiche the Græekes call Isosceles, ισόσκελες. the Latine men æquicurio, and in Englishe tweileke maie the bee called, as in G.H and K. For, thei maie bee of three kyndes, that is to saie, with one square angle, as is G, or with a blunte corner as H, or with all in sharpe corners, as you see in K.

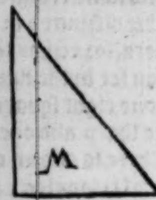
Furthermore, it maie bee that thei haue neuer a one side equall to an other and thei be in three kindes also distincte like the twilekes, as you maie perceiue by these examples M, N. and O, where M, hath a right angle, N. a blunte angle and O, al sharpe angles, these the Grekes and Latine menne doe call scalena, σκαλενόν.



 and

and in Englishe thei maie be called nouelekes, for thei haue no side equall, or like lōg, to any other in ẏ same figure.

Here is to bee noted, that in a triangle, all the angles be called innerangles, except any side bee drawen forthe in length, for then is that fowerth corner called an vtter corner, as in this exaumple because A. B. is drawen in length, therefore the angle C. is called an vtter angle.

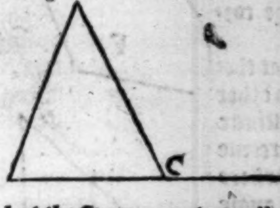


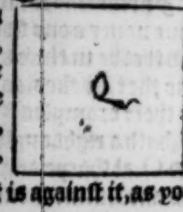
Quadrangle.

And thus haue I doen with triangulled figures, and now foloweth quadrangles, whiche are figures of fower corners, and of fower lines also, of whiche there be diuerse kindes, but chiefly fiue, that is to saie, a square quadrate, whose sides bee all equall, and all the angles square, as you se here in this figure Q. The second kinde is called a long square, whose foure corners bee all Square, but the sides are not equall eche to other, yet is euery side equall to that other that is against it, as you maie perceiue in the figure R.

A square quadrate.

A longe square.

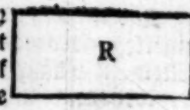




The

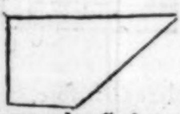
The third kinde is called Losenges, or Diamondes, whose sides be al equall, but it hath neuer a square corner, for twoo of them bee sharpe, and the other twoo bee blunte, as appeareth in S.

A losenge.
A diamonde.



The fowerth sorte are like vnto losenges, saue that thei are longer one waie, & their sides be not equal, yet their corners are like the corners of a losinge, and therfore are thei named Losengelike, or Diamondlike, whose figure is noted with T. Here shall you marke that all those squares, whiche haue their sides all equall, maie bee called also for easie vnderstanding, likesides, as Q. and S. and those that haue onely the contrary sides equall, as R. and T. haue, those will I call likeiammes, for a difference.

A losengelike



The fift sort doeth containe all other fashions of foure cornered figures, and are called of the Greekes Trapezia, of Latine men, mensulæ, and of Arabitians, helmuariphe, thei maie bee called in Englishe borde fourmes, thei haue no side equall [illegible] an other, as these examples shewe, neither kepe thei any rate in their corners, and therefore are thei compted vnruled formes, and thother foure kindes onely are compted ruled formes, in the kinde of quadrangles. Of these vnruled formes there is no nomber, thei are so many & so diuers, yet by art thei maie be chaunged into other kindes of figures, and thereby be brought to measure and proportion, as in the .xii. conclusion is partly taught, but more plainly in my boke of Measuryng you maie see it.

Borde formes

And

And nowe to make an eande of the diuerse kyndes of figures, there dooeth followe now figures of fiue sides, either fiue corners, whiche we maie call cinkangles, whose sides partly are all equall, as in A. and those are coumpted ruled cinkeangles, and partly vnequall, as in B. and thei are called vnruled.

Likewise shall you iudge of siseangles, whiche haue sixe corners, septangles, whiche haue seuen angles, and so foorthe, for as many nombers as there maie bee of sides, and angles, so many diuerse kindes bee there of figures, unto whiche you shall geue names, accordyng to the nomber of their sides and angles, of whiche for this tyme, I will make an eande, and will sette foorthe one example of a siseangle, whiche I had almoste forgotten, and that is it, whose use commeth often in Geometrie, and is called a Squire, is made of twoo long Squares ioyned together, as in this example sheweth.

A squire.

And thus I make an ende to speake of platte fourmes, and will brieflie saie somewhat touchyng the figures of Bodies, whiche partlie haue one platte forme for their bounde, and that iuste round as a Globe hath, or ended long as in an Egge, and a Tunne fourme, whose pictures are these.

The globe as is before.

Howbeit you must marke I meane not the very figure of a Tunne, when I saie tune forme, but a figure like a Tunne, for a Tunne forme

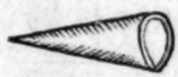
hath

hath but one platte forme, and therefore muſte needes bee rounde at the endes, where as a tunne hath three platte formes, and is flatte at the ende, as partly theſe pictures dooe ſhewe.

Bodies of two plattes, are either cantles or halues of thoſe other bodies, that haue one platte forme, or els thei are like in fourme to twoo ſuche cantles ioyned together, as this A. doeth partly expreſſe: or els it is called a rounde ſpire, or ſtiple forme, as in this figure is ſome what expreſſed.

Nowe of three plattes there are made certaine figures and bodies, as the cantels and halues of al bodies that haue but one plattes, and alſo the halues of halfe globes, and canteles of a globe. Likewiſe a rounde piller, and a ſpire made of a rounde ſpire, ſlit in two partes long waies.

A round ſpire



But as theſe formes bée harde to bée iudged by their pictures, ſo I dooe entende to paſſe them ouer with a greate nomber of other formes of bodies, whiche afterwarde ſhall be ſet forthe in the booke of Perſpectiue, becauſe that without perſpectiue knowledge, it is not eaſie to iudge truely the formes of them in flatte protacture.

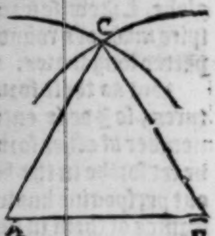
And thus I make an ende for this tyme, of the deffinitions Geometricall, appertainyng to this parte of practiſe, and the reſte will I proſecute as cauſe ſhall ſerue.

The practike woorkyng of sondrie conclusions Geometricall.

¶The firste conclusion.

To make a threlike triangle, or any line measurable.

Take the iuste length of the line with your compasse, and staie the one foote of the compasse, in one of the endes of that line, turnyng the other vp or doune at your wil, drawyng the arche of a circle againste the middle of the line, and doe likewise with the same compasse vnaltered, at the other ende of the line, and where these twoo crooked lines dooeth crosse, from thense drawe a line to eche ende of your firste line, and therefore appere a threelike triangle, drawen on that line.



¶Example.

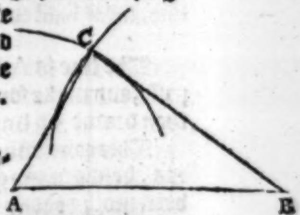
A. B. is the firste line, on whiche I would make the threlike triãgle, therfore I open the cõpasse, as wide as that line is long, and drawe two arche lines that meete in C. then from C. I drawe twoo other lines, one to A. an other to B, and then I haue my purpose.

¶The seconde conclusion.

If you will make a twilik or a nouelike triangle on any certain line.

Consider first the length that you will haue the other sides

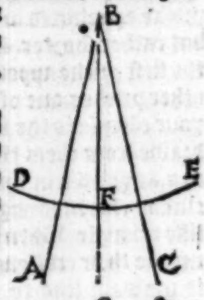
des to containe,and to that lenghth open your compasse and then worke as you did in the threlike triangle,remembring this,that in a nouelike triangle,you muste take two lengthes beside the first line,and drawe an arche line with one of thē at the one ende,the exāple is as the other before.



¶The third conclusion.

To diuide an angle of right lines into twoo equall partes.

First open your cōpasse as largely as you can,so that it doe not extende the length of the shortest line that incloseth the augle.Then set one foote of the compasse in the verie poinct of the angle,and with thother foote drawe a compassed arche frō the one line of the angle to the other,that arche shall you deuide in halfe,& then drawe a line from the angle to the middle of the arche, & so the angle is diuided into ij.equall partes.



¶Example.

Let the triangle be A.B.C. then set I one foote of the compasse in B.and with the other I drawe the arche D.E.which I part into twoo equall partes in F.and then drawe a line from B.to F.and so I haue myne intente.

¶The fourth conclusion.

To deuide any measurable line into twoo equall partes.

Open youre Compasse to the iuste lengthe of the line. And then sette one foote steddelie at the one eande of the line, and with the other foote drawe an arche of a circle againste the middle of the line, bothe ouer it, and also vnder it, then dooe likewaise at the other

ende of the line. And marke where those arche lines doo meete crossewaies, and betwene those twoo prickes drawe a line, and it shall cutte the first line in twoo equall portions.

¶Exaample.

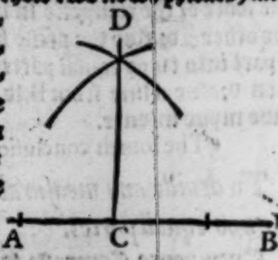
The line is A.B. accordyng to whiche I open the Compasse, and make foure arche lines, whiche méete in C, and D. then drawe I a line from C, so haue I my purpose.

This conclusiõ serueth for makyng of quadrates and squires, beside many other commodities, howbeeit it maie bee doen more readily by this conclusion that foloweth nexte.

¶The.v.conclusion.

To make a plumme line, or any pricke that you will in any right line appoincted.

Open your compas, so that it be not wider then from the pricke appoincted in the line to the shortest ende of the line, but rather shorter. Then set the one foote of the compasse in the first pricke appoincted, and with the other foote marke ij. other prickes, one of eche side of that firste, afterwarde open your cõpasse to the widenes of those two newe prickes, and drawe from them two arche lines, as you did in the firste conclusion, for makyng of a threelike triangle. Then if you doe marke their crossyng, and from it drawe a line to your firste pricke, it shalbe a iust plumline on that place.



¶Eample.

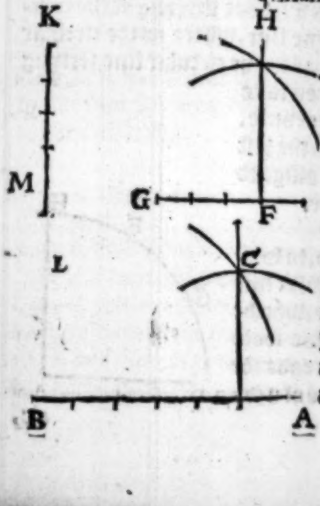
The line is A.B. the pricke on whiche I should make the plumme line, is C. then opẽ I the compasse as wide as A,C. and sette one foote in C, and with the other doe I marke out C. A. and C. B. then open I the compasse as wide as A.B, and make two arche lines whiche doe crosse in D, and so haue I doen.

How bee it, it happeneth sometymes, that the pricke on

whiche

whiche you would make the perpendicular or plumme line, is so nere the ende of your line, that you can not extende any notable length from it to the one ende of the line, and if so be it then that you maie not drawe your line lenger from that ende, then doeth this conclusion require a newe aide, for the last deuise will not serue. In suche case therefore shall you do thus: If your line be of any notable legth, diuide it into fiue partes. And if it bee not so long that it maie yelde fiue notable partes, then make an other line at will, and parte it into fiue equall portions: so that iij of those partes maie be found in your line. Then open your compasse as wide as iij. of these fiue measures be, and set the one foote of the compasse in the pricke, where you would haue your plumme line to lighte (whiche I call the first pricke) & with the other foote drawe an arche line right ouer the pricke, as you can ayme it: then open your compasse as wide as all fiue measure be, and sette the one foote in the fourth pricke, and with the other foote draw an other arche line crosse the first, and where thei two doe crosse, thens drawe a line to the poinct where you would haue the perpendicular line to light, and you haue doen.

¶Example.



The line is A. B. and A. is the pricke, on whiche the perpendiculare line muste light. Therefore I diuide A B. into fiue partes equall, then doe I open the compas to the widenesse of thre partes (that is A. D.) and sette one foote staie in A. and with the other I make an arche line in C. Afterwarde I open the compasse as wide as A. B. (that is as wide as all fiue partes) and set one

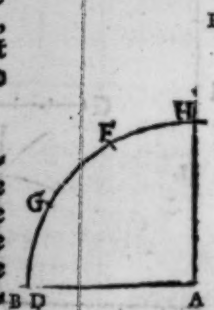
 foote

foote in the fowerth pricke, whiche is E. drawyng an arche line with the other foote in C. also. Then doe I drawe thense a line vnto A, and so haue I doen. But and if the line bee to short, to be parted into fiue partes, I shall deuide it into thre partes onely, as you see the line F.G. and then make D. an other line (as is K.L.) whiche I deuide into fiue suche diuisiōs as F.G. cōtaineth thre, then open I the compasse as wide as fower partes (whiche is K. M.) and so set I one foote of the compasse in F, and with the other I drawe an arche line toward H, then open I the compasse as wide as K.L.) that is all fiue partes) and set one foote in G, (that is the iij. pricke) and with the other I drawe an arche line toward H. also: and where those ij. arche lines doe crosse (whiche is by H.) thence drawe I a line vnto F, and that maketh a verie plumbe line to F.G. as my desire was. The maner of workyng of this cōclusion, is like to the second cōclusiō, but the reason of it doth depende of the .xlvj. proposicion of the first booke of Euclide.

An other waie yet. set one foote of the compasse in the prick, on whiche ye would haue the plumbe line to light, & stretch forthe the other foote toward the longest ende of the line, as wide as you can for the length of the line, & so drawe a quarter of a compasse or more, then without stirryng of the compas, set one foote of it in the same line, where as the circular line did begin, and extēde thother in the circular line, settyng a marke where it doth light, then take half that quantitie more therevnto, and by that pricke that endeth the last part, draw a line to the pricke assigned and it shall be a perpendiculare.

¶Example.

A.B. is the line appointed, to whiche I must make a perpendicular line to light in the pricke assigned, whiche is A. Therefore doe I sette one foote of the compasse in A. and extende the other vnto D, makyng a parte of a circle,

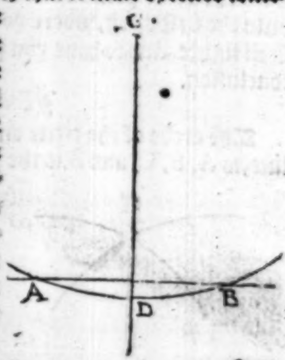


cle, more then a quarter, that is D.E. Then doe I sette one foote of the compasse vnaltered in D. and stretche the other in the circular line, and it doeth light in F. this space betwene D. and F. I deuide into halfe in the pricke G, whiche halfe I take with the compasse, and sette it beyonde F. vnto H. and therefore is H. the pointe, by whiche the perpendicular line must be drawen. so saie I that the line H.A. is a plumbe line to A.B, as the conclusion would.

¶The.vj.conclusion.

To drawe a straighte line frō any pricke that is not in a line, and to make it perpendiculare to an other line.

Open your compasse so wide, that it maie extende somewhat farther, then from the pricke to the line, then sette the one foote of the compasse in the pricke, and with the other shall you draw a compassed line, that shall crosse that other firste line in twoo places. Now if you deuide that arche line into twoo equall partes, and frō the middle pricke therof vnto the prick without the line, you drawe a straight line, it shalbe a plumbe line to that first line, accordyng to the conclusion.



¶Example.

C. is the appointed pricke, from whiche vnto the line A. B. I must drawe a perpendicular. Therfore I open the compasse so wide, that it maie haue one foote in C. and the other to reache ouer the line, and with that foote I drawe an arch line, as you se betwene A. and B, whiche arche line I deuide in the middle in the poincte D. Then drawe I a line from C. to D, and it is perpendiculare to the line A. B, accordyng as my desire was.

The

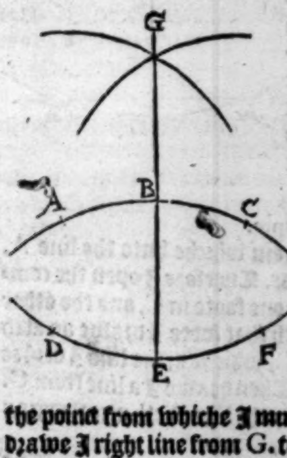
¶The.vij.conclusion.

To make a plumbe line or any portion of a circle, and that on the vtter or inner bught.

Marke first the prick where the plumbe line shall light: and pricke out on eche side of it two poinctes equally distante from that first pricke. Then set the one foote of the compas in one of those side prickes, and the other foote in the other side prick, and first moue on of the feete, and drawe an arche line ouer the middle pricke, then set the compas steddie with the one foote in the other side pricke, and with the other foote drawe an other arche line, that shall cut that firste arche, and from the verie poincte of their meetyng, drawe a right line vnto the first prick, where you doe minde that the plūbe line shall lighte. And so haue you perfourmed the intente of this conclusion.

¶Example.

The arche of the circle on whiche I would erecte a plūbe line, is A. B. C. and B. is the pricke where I would haue the plumbe line to light. Therefore I meate out twoo equall distaunces on eche side of that pricke B. and thei are A. C. Then open I the Compas as wide as A.C. and settyng one of the feete in A. with the other I drawe an arche line, whiche goeth by G. Likewayes I set one foote of the compas steddily in C. and with the other I drawe an arche line, goyng by G. also. Now consideryng that G. is the pricke of their meetyng, it shall bée also the poinct from whiche I must draw the plumbe line. Then drawe I right line from G. to B. and so haue myne intente.



Now

Nowe as A. B. C. hath a plumbe line erected on his vtter bught, so maie I erect a plūbe line on the inner bught of D.E F, doyng with it as I did with the other, that is to saie, firste settyng forthe the pricke where the plumbe line shall light, whiche is E, and then makyng one other on eche side, as are D. and F. And then proceding as I did in the example before.

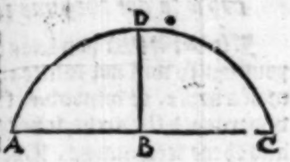
¶The. viij. conclusion.

How to deuide the arche of a circle into twoo equall partes, without measuryng the arche.

Deuide the corde of that line into twoo equall portions, and then from the middle pricke, erecte a plumbe line, and it shall parte that arche in the middle.

¶Example.

The arche to be deuided is A. D. C. the corde is A. B. C. this corde is deuided in the middell with B. from whiche pricke if I erecte a plumbe line as A. B. D. then will it deuide the arche in the middle, that is to saie, in D.



¶The. ix. conclusion.

To doe the same thyng otherwise. And for shortnes of worke, if you will make a plumbe line without much labor, you maie do it with your squire, so that it be iustly made, for if you applie the edge of the squire to the line in whiche the pricke is, and foresee the very corner of the squire do touche the pricke. And then from that corner, if you drawe a line by the other edge of the squire, it will be a perpendiculare to the former line.

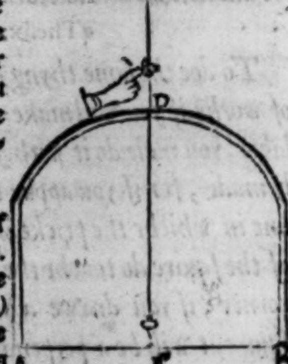
¶Example.

A. B. is the line, on whiche I would make the plumbe line or perpendiculare. And therefore I make the pricke, from whiche the plumbe line muste rise, whiche here is C. Then do I sette one edge of my squire (that is B.C.) to the line A. B, so that the corner of the squire doe touche C. iustly. And frō C. I drawe a line by the other edge of the squire, (which is C.D.) And so haue I made the plumbe line D. C, whiche I sought for.

¶The.x.conclusion.

How to doe the same thyng an other waie yet.

If so bee it that you haue an arche of suche greatnes, that your squire will not suffice thereto, as the arche of a bridge, or of a house, or windowe, then maie you doe this. Meete vnderneath the arche, where the middle of his corde will be, and there set a marke. Then take a long line with a plummet, and holde the line in suche a place of the arche, that the plummet doe hange iustly ouer the middle of the corde, that you did deuide before, and then the line dooeth shewe you the middle of the arche.



¶Example.

The arche is A. D. B. of whiche I trie the middle thus. I drawe a corde from one side to the other (as here is A. B,) whiche I deuide in the middle in C. Then take I a line with a plummette (that is D, E,) and so holde I the line, that the plummet

plummet

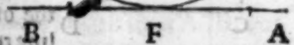
plummet E. doeth hange ouer C. And then I saie that D, is the middle of the arche. And to the intent that my plummet shall poincte the more iustly, I doe make it sharpe in the nether ende, and so maie I trust this worke for certaine.

¶ The.xj.conclusion.

When any line is appoincted and without a pricke, whereby a parallele must bee drawen, how you shall dooe it.

Take the iust measure betwene the line and the pricke, accordyng to whiche you shall open your Compasse. Then pitche one foote of your compasse, at the one ende of the line, and with the other foote drawe a bowe line, right ouer the pitche of the compasse, likewise dooe at the other ende of the line, then drawe a line that shal touche the uttermoste edge of bothe those bowe lines, and it will bee a true parallele to the first line appoincted.

¶ Example.



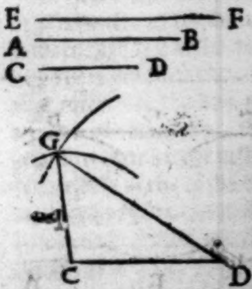
A. B. is the line unto whiche I muste drawe an other gemowe line, whiche muste passe by the pricke C, firste I meate with my compasse the smallest distaunce that is frõ C. to the line, and that is C, F, wherefore staiyng the Compasse at that distaunce, I sette one foote in A, and with the other foote I make a bowe line, whiche is D, then likewise sette I the one foote of the compasse in B, and with the other I make the seconde bowe line, whiche is E. And then drawe I a line, so that it toucheth the uttermoste edge of both these bowe lines, and that line passeth by the pricke C., ende is a gemowe line to A.B. as my seekyng was.

¶The.xij.conclusion.

To make a triangle of any three lines, so that the lines bee suche, that any twoo of them bee longer then the thirde. For this rule is generall, that any twoo sides of enery triangle taken together, are longer then the other side that remaineth.

If you doe remember the firste and seconde conclusions, then is there no difficultie in this, for it is in maner the same worke. Firste consider the three lines that you must take, and sette one of them for the grounde line, then worke with the other twoo lines, as you did in the firste and seconde conclusions.

¶Example.



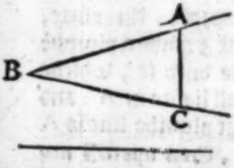
I haue three lines. A. B. and C. D. and E. F. of whiche I putte C. D. for my grounde line, then with my Compasse I take the lengthe of A. B. and sette the one foote of my Compasse in C. and drawe an arche line with the other foote. Likewaies I take the length of E. F. and set one foote in D, and with the other foote I make an arche line crosse the other arche, and the pricke of their meetyng (whiche is G.) shall bee the thirde corner of the triangle, for in all suche kyndes of workyng to make a triangle, if you haue one line drawen, there remaineth nothyng els but to finde where the pricke of the thirde corner shall bee, for twoo of them must needes bee at the twoo endes of the line that is drawen.

¶The.xiij.conclusion.

If you haue a line appoincted, and a pointe in it limited, howe you maie make on it a right lined angle, equall to an other right lined angle, all ready assigned.

Firste drawe a line againste the corner assigned, and so is it a triangle, then take heede to the line, and the pointe in it assigned, and consider if that line from the pricke to this end bee as longe as any of the sides that make the triangle assigned, and if it bee longe enough, then pricke out there the length of one of the lines, and then woorke with the other twoo lines, accordyng to the laste conclusion, makyng a triangle of three like lines, to that assigned triangle. If it bee not long enough, then lengthen it firste, and afterwarde doe as I haue saied before.

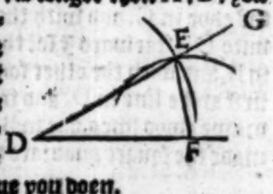
¶Eample.



Lette the angle appointed be A.B.C. and the corner assigned, B. Farthermore lette the limitted line bee D.G, and the pricke assigned D.

Firste therefore by drawyng the line A.C. I make the triangle A.B.C.

Then consideryng that D.G. is longer then A.B. you shall cutte out a line from D. towarde G, equall to A.B, as for example D.E. Then measure out the other twoo lines, and woorke with them, accordyng as the conclusion with the firste also and the seconde teacheth you, and then haue you doen.



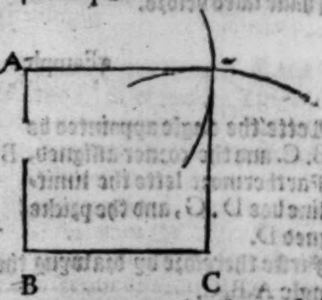
¶ The, xiiij. conclusion.

To make a square quadrate of any light line appointed.

Firste make a plumbe line unto youre line appoincted, whiche shall light at one of the endes of it, accordyng to the fifth conclusion, and let it be of like length as your firste line is, then open your compasse to the iuste length of one of the, and set one foote of the compasse in the ende of the one line, and with the other foote drawe an arche line, there as you thinke that the fowerth corner shalbe, after that set the one foote of the same compasse unsturred, in the ende of the other line, and drawe an other arche line crosse the first arche line, and the poincte that thei dooe crosse in, is the pricke of the fowerth corner of the square quadrate whiche you seeke for, therefore drawe a line from that pricke to the ende of eche line, and you shall thereby haue made a square quadrate.

¶ Example

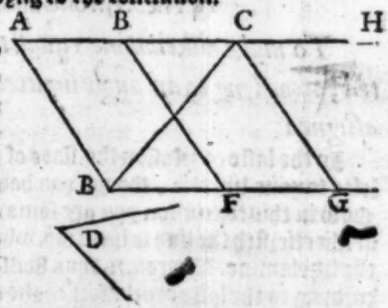
A. B. is the line proposed, of whiche I shall make a Square quadrate, therefore, first I make a plumbe line unto it, whiche shall lighte in A. and that plumbe line is A C, then open I my compasse as wide as the length of A. B. or A. C. (for thei must be bothe equall) and I set the one foote of the ende in C, and with the other I make an arche line nigh unto D, afterward I set the compasse againe with one foote in B, and with the other foote I make an arche line crosse the first arche line in D, and from the pricke of their crossyng, I drawe twoo lines, one in B, and an other to C, and so haue I made the square quadrate that I intended.



¶ The, xv. conclusion,

To make a likeiamme equall to a triangle appointed, and that in a right lined angle limitted.

Firste from one of the angles of the triangle, you shall drawe a gemowe line, whiche shalbe a parallele to that side of the triangle, on whiche you will make that likeiamme. Then on one ende of the side of the triangle, whiche lieth against the gemowe line, you shall drawe forthe a line unto the gemowe line, so that one angle that commeth of those .ij. lines be like to the angle, whiche is limitted unto you. Then shall you deuide into twoo equall partes, that side of the triangle, whiche beareth that line, and from the pricke of that deuision, you shall raise an other line parallele to that former line, and continewe it unto the firste gemowe line, and then of those twoo laste gemowe lines, and the first gemow line, whiche is the halfe side of the triangle, is made alikeiamme equall to the triangle appoincted, and hath an angle like to an angle limitted, accordyng to the conclusion.



Example.
B.C.G, is the triangle appoincted unto, whiche I muste make an equall likeiamme. And D, is the angle that ẏ likeiāme muste haue. Therefore firste intendyng to erecte the likeiāme on the one side, that the grounde line of the triangle (whiche is B.G.) I dooe drawe a gemowe line by C, and make it parallele to the ground line B.G, and that newe gemow line is A.H. Then doe I raise a line from B. unto the gemowe line, (which line is A.B.) and make an angle equall to D, that is the appointed angle (accordyng as theight conclusion teacheth, and that angle is B.A.E. Then to proceade, I doe parte in the middle

the saied ground line B.G, in the pricke F. from which pricke I drawe to the firste gemowe line (A.H.) an other line that is parallele to A.B, and that line is E.F. Now saie I that the likeiamme B.A.E.F, is equall to the triangle B.C.G. And also that it hath one angle (that is B.A.E.) like to D. the angle that was limitted. And so haue I myne intente. The proofe of the equalnesse of those twoo figures, doeth depende of the .xlj. proposition of Euclides firste booke, and is the .xxxj. proposition of this seconde booke of Theoremes, whiche saieth, that when a triangle and a likeiamme, bee made betwene twoo self same gemowe lines, and haue their grounde line of one length, then is the likeiamme double to the triangle, whereof it followeth, that if twoo suche figures so drawen differ in their grounde line onely, so that the grounde line of the likeiamme be but halfe the grounde line of the triangle, then bee those twoo figures equall, as you shall more at large perceiue by the booke of Theoremes, in the .xxxj. Theoreme.

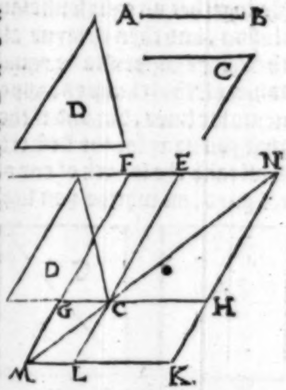
¶ The .xvj. conclusion.

To make a likeiamme equall to a triangle appointed, accordyng to an angle limitted, and on a line also aßigned.

In the laste conclusion the sides of your likeiamme were lefte to your libertie, though you had an angle appoincted. Now in this conclusion you are somewhat more restrained of libertie, sith the line is limitted, whiche must be the side of the likeiamme. Therefore thus shall you procede. First accordyng to the laste conclusion, make a likeiamme in the angle appoincted, equall to the triangle that is assigned. Then with your compasse take the length of your line appoincted, and set out twoo lines of the same length in the seconde gemow lines, beginnyng at the one side of the likeiamme, and by those twoo prickes shall you drawe an other gemowe line, whiche shall be parallele to twoo sides of the likeiamme. Afterward shall you drawe twoo lines more, for the accomplishement

plishemente of your worke, whiche better shall bee perceiued by a shorter example, then by a greater number of wordes, onely without example, therefore I will by example set forthe the whole worke.

¶Example.



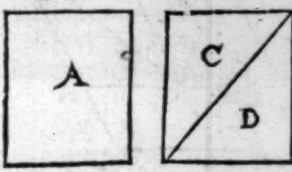
First, accordyng to the laste conclusion, I make the likeiamme E.F.C.G, equall to the triangle D, in the appoincted angle, whiche is E. Then take I the length of the assigned line (whiche is A.B,) and with my compasse I sette forth the same lengthe in the twoo gemowe lines N.F, and H.G, settyng one foote in E, and the other in N, and againe settyng one foote in C, and the other in H. Afterwarde I drawe a line from N. to H, whiche is a gemowe line, to twoo sides of the likeiamme, then drawe I a line also frō N. unto C, and extende it untill it crosse the lines E.L. and F.G, whiche bothe must bee drawen forthe longer then the sides of the likeiamme, and where that line doeth crosse F.G there I set M. Nowe to make an ende, I make an other gemow line, whiche is a parallele to N.F. and H.G, and that gemow line doeth passe by the pricke M, and then haue I doen. Now saie I that H.C,K.L, is alikeiamme equall to the triangle appoincted, whiche was D, and is made of a line assigned, that is A.B, for H.C, is equall unto A.B, and so is K.L. The proofe of the equalnes of this likeiamme unto the triāgle, depēdeth of the .xxxij. Theoreme: as in the boke of Theoremes doeth appeare, where it is declared, that in all likeiammes, whē there are more then are made about one bias line, the fillsquares of euery of them must needes bee equall.

 ¶The

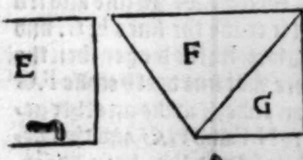
¶The.xxij.conclusion.

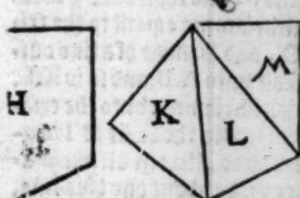
To make a likeiamme equall to any right lined figure, and that on an angle appoincted.

The readiest waie to worke this conclusion, is to tourne that right lined figure into triangles, and then for euery triãgle together an equall likeiamme, accordyng vnto the.xj.cõclusion, and then to ioyne all those likeiammes into one, if their sides happen to be equall, whiche thyng is euer certain, when all the triangles happen iustely betwene one paire of gemowe lines, but and if thei will not frame so, then after that you haue for the first triangle made his likeiamme, you shall take the length of one of his sides, and set that as a line assigned, on whiche you shall make the other likeiammes, according to the xij.conclusion and so shall you haue all your likeiames with twoo sides equall, and twoo like angles, so that you maye easilie ioyne them into one figure.



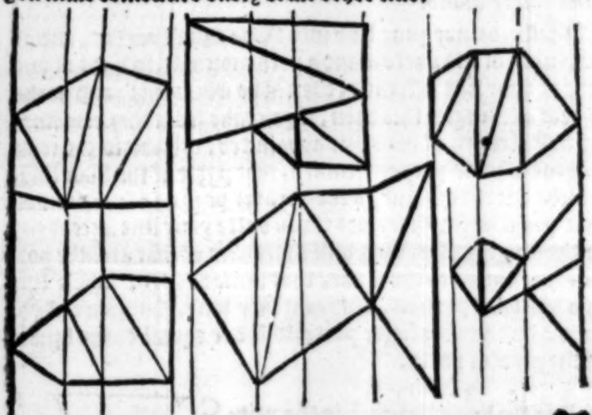
¶Example.





If the right lined figure bée like vnto A, then maie it bee tourned into triangles, that will stande betwene twoo paralleles aniwaies, as you may see by C, and D, for twoo sides of bothe the triangles are paralleles. Also if the right lined figure be like vnto E. then will it bee tourned into triangles, liyng betwene twoo paralleles also, as the other did before, as in the example of F, G. But and if the right lined figure bee like vnto H, and so tourned into triangles, as

you see in K.L.M, where it is parted into.iij.triangles, then will not all those triangles lye betwene one paire of paralleles, or gemowe lines, but must haue many, for euery triangle muste haue one paire of paralleles seuerall, yet it maie happen that when there bee three or fower triangles, twoo of thē maie happen to agre to one paire of paralleles, whiche thyng I remitte to euery honest witte to serche, for the maner of their draught will declare, how many paires of paralleles thei shall neede, of whiche varietie, because the examples are infinite, I haue set forthe these fewe, that by them you maie coniecture duely of all other like.



Further explication you shall not greatly nede, if you remember what hath been taughte before, and then diligently beholde, how these sundrie figures be tourned into triangles. In the first, you see I haue made fiue triangles, and fower paralleles, in the seconde seuen triangles, and fower paralleles, in the thirde three triangles, and fiue paralleles: In the fowerth you see fiue triangles and fower paralleles: In the fift, fower triangles, and fower paralleles, and in the sixt there are fiue triāgles, and fower paralleles. Howbeit a man maie at libertie alter them into diuers formes of triangles,

 and

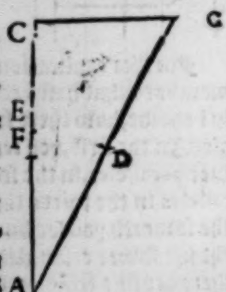
and therefore I leaue it to the discretion of the woorkemaister, to dooe in all suche cases as he shall thinke beste, for by these examples (if thei bee well marked) maie al other like conclusions bee wrought.

¶The,xviij.conclusion.

To parte a line aßigned after suche a sorte, that the square that is made of the whole line, and one of his partes, shalbe equall to the square that commeth of the other parte alone.

Firste, deuide your line into twoo equall partes, and of the length of one parte make a perpendicular, to light at one ende of your line assigned, then adde a bias line, and make thereof a triangle, this doen, if you take from this bias line, the halfe length of your line appoincted, whiche is the iuste lengthe of your perpendiculare, that parte of the bias line, whiche doeth remaine, is the greater portion of the diuision that you seke for: therefore if you cutte your line, accordyng to the lengthe of it, then will the square of that greater portion, bee equall to the square that is made to the whole line and his lesser portion. And contrary wise, the square of the whole line and his lesser parte, will bee equall to the square of the greater parte.

¶Example.



A.B. is the line assigned E. is the middle pricke of A. B, B.C. is the plumbe line or perpẽdicular, made of the half of A.B, equall to A.E, either B. E, the bias line is C. A, from whiche I cut a pece, that is C. D. equall to C.B, & accordyng to the length of the pece that remaineth (whiche is D.A,) I doe deuide the line A. B, at whiche deuision I set E. Now saie I, that this line A.B (whiche was assigned vnto me) is so deuided in this poinct F, that

that

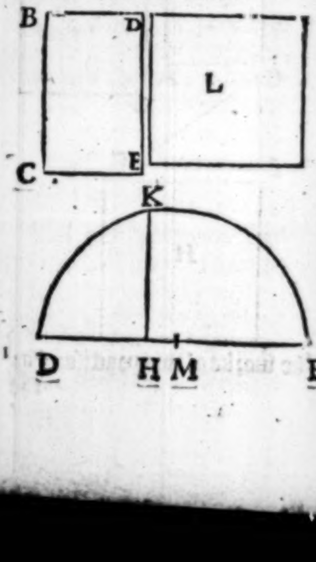
that the square of the whole line A.B, and of the one portion (that is F. B, the lesser part) is equall to the square of thother part, which is F. A, & is the greater part of the first line The profe of this equalitie shall you learne by the .xl. Theoreme.

¶ The.xix.conclusion.

To make a square quadrate equall to any right lined figure appoincted.

First make a likeiamme equall to that right lined figure with a right angle, accordyng to the .xi. conclusion, then consider the likeiamme, whether it haue all his sides equall, or not: for if thei be all equall, then haue you doen your conclusion, but and if the sides be not all equall, then shal you make one right line, iuste as long as ii. of those vnequall sides, that line shall you deuide in the middle, and on that pricke drawe half a circle, then cut from that diameter of the halfe circle a certain portion, equall to the one side of the likeiamme, and from that poinct of diuision shall you erecte a perpendicular, whiche shall touche the edge of the circle. And that perpendicular shalbe the iust side of the square quadrate, equall bothe to the likeiāme, and also to the right lined figure appointed, as the conclusion willed.

¶ Example.



K. is the right lined figure appoincted, and B.C. D. E, is the likeiamme, with right angles equall vnto K, but because that this likeiamme is not a square quadrate, I muste tourne it into suche one after this sorte, I shall make one righte line, as long as twoo vnequall sides of the likeiamme, that line here is F. G, whiche is equall to B. C, and C. E.

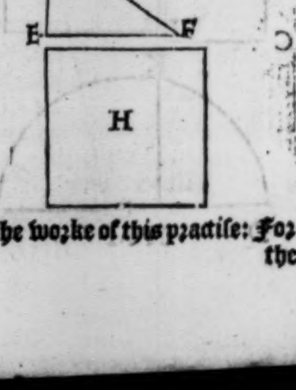
Then parte I that line in the middle in the pricke M, and on that pricke I make halfe a circle, accordyng to the length of the diameter F. G. Afterwarde I cutte awaie a peece from F, G, equall to C. E, markyng that poincte with H. And on that pricke I erecte a perpendicular H. K, whiche is the iust side to the square quadrate that I seeke for, therefore accordyng to the doctrine of the tenth conclusion, of that line I doe make a square quadrate, and so haue I attained the practise of this conclusion.

¶The.xx.conclusion.

VVhen any twoo square quadrates are set forthe, how maie you make one equall to them bothe.

Firste, drawe a right line equall to the side of one of the quadrates: and on the eande of it make a perpendiculare, equall in length to the side of the other quadrate, then drawe a bias line betwene those twoo lines, makyng thereof a right angeled triangle. And that bias line will make a square quadrate, equall to the other twoo quadrates appoincted.

¶Example.



A. B. and C. D, are the twoo square quadrates appoincted, vnto which I must make one equal square quadrate. First therefore I doe make a right line E. F, equal too one of the sides of the square quadrate A. B. And on the one ende of it I make a plumbe line E. G, equall to the side of the other quadrate D. C. Then drawe I a bias line. G. F, whiche beyng made the side of a quadrate (accordyng to the teth conclusion) will accomplishe the worke of this practise: For the

the quadrate H,is as muche iuste as the other twoo,I meane A,B,and D,C.

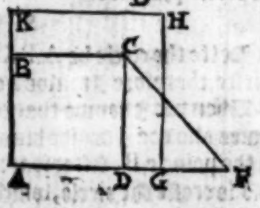
¶The.xxj.conclusion.

VVhen any twoo quadrates bee sette foorthe,howe to make a squire aboute the one quadrate,whiche shall bee equall to the other quadrate.

Determine with your self,aboute whiche quadrate you will make the Squire, and drawe one side of that quadrate foorthe in length,accordyng to the measure of the side of the other quadrate, whiche line you maie call the grounde line, and then haue you a righte angle made on this line,by an other side of thesame quadrate:Therefore tourne that into a right cornered triangle, accordyng to the woorke in the last conclusion,by makyng of a bias line, and that bias line will perfourme the woorke of youre desire. For if you take the lengthe of that bias line with your compasse, and then sette one foote of the Compasse in the farthest angle of the firste quadrate(whiche is the one ende of the grounde line)and extende the other foote on the same line,accordyng to the measure of the bias line, and of that line make a quadrate, enclosyng the first quadrate, then will there appeare the forme of a squire aboute the first quadrate, whiche squire is equall to the ij quadrate.

¶Example.

The first square quadrate is A.B.C.D,and the seconde is E. Nowe would I make a Squire aboute the quadrate A,B,C,D, whiche shall bee equall unto the quadrate E



Therefore

Therefore firste I drawe the line A.D, more at length, according to the measure of that side of E, as you see, from D. vnto F, and so the whole line of bothe these seuerall sides is A.F, then make I a bias line from C, to F, whiche bias line is the measure of this worke. Wherefore I open my compasse, accordyng to the length of that bias line C.F, and sette the one Compasse foote in A, and extende the other foote of the compasse towarde F, makyng this pricke G, from which I erecte a plumbe line G.H, and so make out the square quadrate A. G. H. K, whose sides are equall eche of them in A. G. And this square doeth containe the first quadrate. A.B.C. D, and also a squire G.H. K, whiche is equall to the seconde quadrate E, for as the last conclusion declareth, the quadrate A.G.H.K, is equall to bothe the other quadrates proposed, that is A.B.C, D, and E. Then muste the squire G.H.K, nedes be equall to E, consideryng that all the rest of that great quadrate, is nothyng els but the quadrate self, A. B. C. D, and so haue I the intente of this conclusion.

¶The.xxj.conclusion.

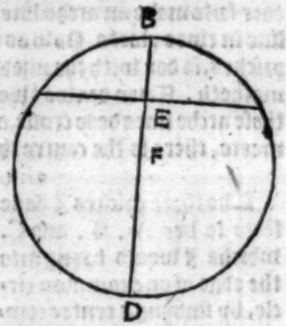
To finde out the centre of any circle aßigned.

Drawe a corde or stryng line crosse the circle, then deuide into twoo equall partes, bothe that corde, and also the bowe line, or arche line, that serueth to that corde, and from the prickes of those deuisions, if you drawe an other line, crosse the circle, it must nedes passe by the centre. Therefore diuide that line in the middle, and the middle pricke is the centre of the circle proposed.

¶Example.

Lette the circle be A.B.C.D, whose centre I shall seeke. Firste, therefore I drawe a corde crosse the circle, that is A. C. Then doe I deuide that corde in the middle, in E, and likewaies also doe I deuide his arche line A.B.C, in the middle, in the poincte B, Afterwarde I drawe a line from B, to E, and so crosse the circle, whiche line is B, D, in whiche line is the

the centre that I seeke for. Therefore if I part that line B.D. in the middle into twoo equall portions, that middle pricke (whiche here is F.) is the verie centre of the saied circle that I seeke. This conclusion maie other waies bee wrought, as the moste parte of conclusions haue sunderie fourmes of practise, and that is, by makyng three prickes in the circumference of the circle at libertie where you will, and then findyng the



centre of those three prickes, whiche woorke because it serueth for sondrie vses, I thinke meete to make it a seuerall conclusion by it self.

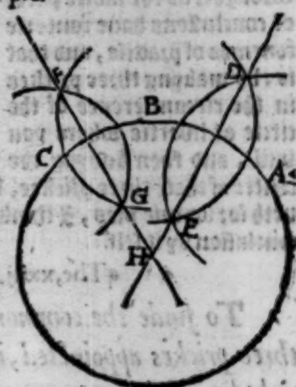
¶The.xxiij.conclusion.

To finde the common centre belongyng to any three prickes appoincted, if thei bee not in an exacte right line.

It is to bee noted, that though euery small arche of a greater circle doe seem to bee a right line, yet in verie dede it is not so, for euery parte of the circumference of all circles is compassed, though in little arches of great circles, the eye can not discerne the crookednesse, yet reason dooeth alwaies declare it, therefore three prickes in an exacte right line, can not bee brought into the circumference of a circle. But and if thei bee not in a right line, howe so euer thei stande, thus shall you finde their common centre. Open your Compasse so wide, that it bee somewhat more then the halfe distaunce of twoo of those prickes. Then set the one foote of the compasse in the one pricke, and with the other foote drawe an

arche line towarde the other pricke. Then againe putte the foote of your compasse in the seconde pricke, and with the other foote make an arche line, that maie crosse the first arche line in twoo places. Now as you haue doen with those twoo prickes, so doe with the middle pricke, and the thirde that remaineth. Then drawe twoo lines by the poinctes, where those arche lines dooe crosse, and where those twoo lines doe meete, there is the centre that you seeke for.

¶Example.



The three prickes I haue sette to bee A. B. and C. whiche I would bryng into the edge of one common circle, by findyng a centre common to them all, first therefore I open my compasse, so that thei occupie more then the halfe distaunce betwene twoo prickes (as are A. B.) and so settyng one foote in A. and extendyng the other toward B, I make the arche line D. E. Likewise settyng one foote in B. and turnyng the other toward A. I drawe an other arche line, that crosseth the firste D. and E. Then from D to E, I drawe a right line D. H. After this I open my compasse to a newe distance and make twoo arche lines betwene B. and C, whiche crosse one the other in F. and G, by whiche twoo poinctes I drawe an other line, that is F. H. And because that the line D. H, and the line F. H. doe meete in H, I saie that H, is the centre that serueth to those three prickes. Now therefore if you set one foote of your compasse in H, and extende the other to any of the three prickes, you maie drawe a circle whiche shall enclose those three prickes in the edge of his circumference, and thus haue you attained the vse of this conclusion.

The

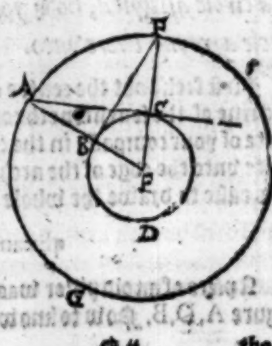
¶The.xxiiij.conclusion.

To drawe a touche line vnto a circle, from any poincte aßigned.

Here must you vnderstande, that the pricke must be with out the circle, els the conclusion is not possible. But the prick or poinct beyng without the circle, thus shall you procede: open your compasse, so that the one foote of it maie bee set in the centre of the circle, and the other foote on the pricke appoincted, and so drawe an other circle of that largenesse about the same cētre: and it shall gouerne you certainly in makyng the saied touche line. For if you drawe a line from the pricke appoincted, vnto the centre of the circle, and marke the place where it doeth crosse the lesse circle, and from that poincte erecte a plumbe line, that shall touche the edge of the vtter circle, and marke also the place wher that plumbe line crosseth that vtter circle, and from that place drawe an other line to the centre, takyng heede where it crosseth the lesser circle, if you drawe a plumbe line from that pricke, vnto the edge of the greater circle, that line I saie is a touche line, drawyng from the poincte assigned, accordyng to the meanyng of this conclusion.

¶Example.

Lette the circle bee called B,C,D, and his centre E, and the pricke assigned A, open your Compasse now of suche wideneße, that the one foote maie bee sette in E, whiche is the Centre of the circle, and the other in A, whiche is the poincte assigned, and so make an other greater circle (as here is A,F,G,) then drawe a line frō A. vnto E, and where that line dooeth crosse the inner circle (whiche here is in



the pricke B.) there erecte a plumbe line vnto the line A. E. and let that plumbe line touche the vtter circle, as it doth here in the poincte F. so shall B.F. be that plumbe line. Then from F. vnto E. drawe an other line, whiche shall bee F.E. and it will cutte the inner circle, as it dooeth here in the poincte C. from whiche poincte C. if you erecte a plumbe line vnto A. then is that line A, C. the touche line, whiche you should finde. Notwithstandyng that this is a certaine waie to finde any touche line, and a demonstrable fourme, yet more easelily manifold maie you finde, and make any suche line with a true ruler, laiyng the edge of the ruler, to the edge of the circle, and to the pricke, and so drawyng a righte line, as this exãple sheweth wher the circle is E. the prick assigned is A. and the ruler C. D. by whiche the touche line is drawen, and that is A.B. and as this waie is light to doe, so is it certain enough for any kinde of workyng.

¶The.xxv.conclusion.

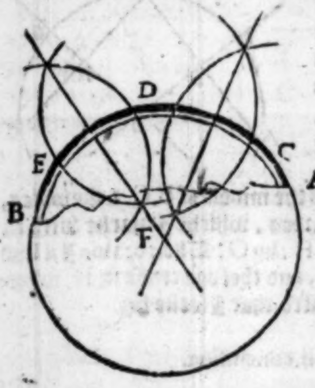
When you haue any peece of the circumference of a circle aßigned, how you maie make out the whole circle agreyng there vnto.

First seeke out the centre of that arche, accordyng to the doctrine of the seuententh conclusion, and then settyng one foote of your compasse in the centre, and extedyng the other foote vnto the edge of the arche, or peece of the circũference, it is easie to drawe the whole circle.

¶Example.

A pẽece of an old piller was founde, like in fourme to this figure A, D, B. Now to knowe howe muche the compasse of the

the whole piller was, seyng by this parte it appeareth, that it was rounde, thus shall you dooe. Make in a table the like draught of the circumference by the self patron, vsyng it as it were a crooked Ruler. Then make thre prickes in that arche line, as I haue made, C. D. and E. And then finde out the common centre to theim all, as the seuentene conclusion teacheth. And that centre is here F. now to settyng one foote of your compasse in F. and the other in C. D, either in E, and so makyng a compasse, you haue your whole intent.



¶The.xxvj.conclusion.

To finde the centre to any arche of a circle.

If so be it that you desire to finde the centre by any other waie, then by those thre prickes, consideryng that sometymes you can not haue so muche space in the thing, where the arche is drawen, as should serue to make those fower bowe lines, then shall you dooe thus: Parte that arche line into twoo partes, equall either vnequall, it maketh no force, and vnto eche portion drawe a corde, either a strynge line. And then accordyng as you did in one arche in the sixteneth conclusion, so dooe in bothe those arches here, that is to saie, deuide the arche in the middle, and also the corde, and drawe then a line by those twoo diuisions, so then are you sure that that line goeth by the centre. Afterwarde do likewaies with the other arche and his corde, and where those twoo lines do crosse, there is the centre that you seeke for.

 ¶Example

¶Example.

The arche of the circle A.B.C.vnto whiche I must seke a centre, therefore first I doe deuide it into twoo partes, the one of them is A.B, and the other is B.C. Then doe I cutte euery arche in the middle, so is E, the middle of A.B, and G, is the middle of B.C. Likewaies, I take the middle of their cordes, whiche I marke with F, and H, settyng F. by E, and H. by G. Then drawe I a line from E. to F. and frō G. to H, and thei doe crosse in D. wherfore saie I, that D, is the centre, that I seeke for.

¶The, xxvij. conclusion.

To drawe a circle within a triangle appoincted.

For this conclusion and all other like, you muste vnderstande, that when one figure is named to bee within an other, that is not otherwaies to bee vnderstande, but that either euery side of the inner figure, dooeth touche euery corner of the other, either els euery corner of the one, dooeth touche euery side of the other. So I call that triangle drawen in a circle, whose corners dooe touche the circumference of the circle. And that circle is contained in a triangle, whose circumference dooeth touche iustely euery side of the triangle, and yet dooeth not crosse ouer any side of it. And so that quadrate is called properly, to bee drawen in a circle, when all his fower angles dooeth touche the edge of the circle. And that circle is drawen in a quadrate, whose circumference dooeth touche euery side of the quadrate, and likewaies of other figures.

Examples

Examples are these. A.B.C.D.E.F.

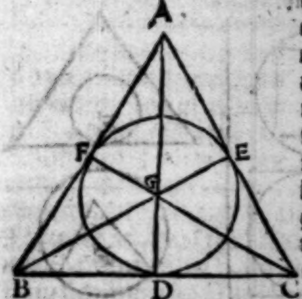
A.is the circle in a triangle. C.a quadrate in a circle.

B.a triangle in a circle. D.a circle in a quadrate.

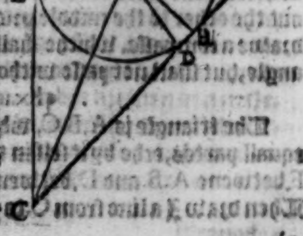
In these twoo laste figures E.and F, the circle is not named, to bee drawen in a triangle, because it doeth not touche the sides of the triangle, neither is the triangle coumpted to be drawen in the circle, because one of his corners doeth not touche the circumference of the circle, yet (as you se) the circle is within the triangle, and the triangle within the circle, but neither of thē is properly named to be in thother. Nowe to come to the conclusion. If the triangle haue all three sides like, then shall you take the middle of euery side, and frō the contrary corner drawe a righte line vnto that poincte, and where those lines doe crosse one an other, there is the cētre. Then set one foote of the compasse in the centre, and stretch out the other to the middle pricke of any of the sides, and so drawe a compasse, whiche shall touche euery side of the triangle, but shall not passe without any of them.

Example.

The triangle is A,B,C, whose sides I doe part into twoo equall partes, eche by it self in these poinctes D.E.F. putting F. betwene A.B. and D. betwene B.C. and E. betwene A.C. Then draw I a line from C. to F. and an other from A. to D



and the third from B. to E. And where all those lines do mete (that is to saie M.G.) I set the one foote of my Compasse, because it is the common centre, and so drawe a circle accordyng to the distaunce of any of the sides of the triangle. And then finde I that circle to agree iustly to all the sides of the triangle, so that the circle is iustely made in the triangle, as the conclusion did purporte. And this is euer true, when the triangle hath all three sides equall, either at the least twoo sides like long. But in thother kindes of triangles you must deuide euery angle in the middle, as the thirde conclusion teacheth you. And so drawe lines from eche angle to their middle pricke. And where those lines doe crosse, there is the common centre, from whiche you shall drawe a perpendiculare to one of the sides. Then sette one foote of the compasse in that centre, and stretch the other foote, accordyng to the length of the perpendiculare, and so drawe your circle.



¶Example.

The triangle is A. B. C. whose corners I haue deuided in the middell with D. E, F, and haue drawen the lines of deuision A, D, B, E, and C, F, whiche crosse in G, therefore shall G, bee the common centre. Then make I one perpendiculare from G vnto the side A. C. and that

is G.H. Now sette I one foote of the compasse in G.and extende the other foote vnto H.and so drawe a cōpasse, whiche will iustly aunswere to that triangle, accordyng to the meanyng of the conclusion.

¶The.xxviij.conclusion.

To drawe a circle about any triangle aßigned.

Firste deuide twoo sides of the triangle equally in halfe, and from those twoo prickes erecte twoo perpendiculares, whiche must nedes meete in crosse, and that poincte of their meetyng is the centre of the circle that muste bee drawen, therefore sette one foote of the compasse in that poincte, and extende the other foote to one corner of the triangle, and so make a circle, and it shall touche all three corners of the triangle.

¶Example.

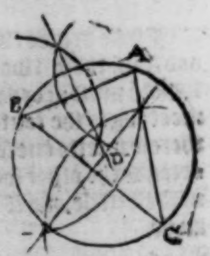
A,B,C, is the triangle, whose twoo sides A.C. and B.C. are deuided into twoo equall partes in D. and E. settyng D. betwene B, and C. and E. betwene A. and C. And from eche of those twoo poinctes is there erected a perpendiculare (as you see D.F. and E.F.) whiche meete, and crosse in F, and stretche forthe the other foote of any corner of the triãgle, and so make a circle, that circle shall touche euery corner of the triangle, and shall enclose the whole triangle, accordyng as the conclusion willeth.

¶An other waie to doe the same.

And yet an other waie maie you doe it, accordyng as you

learned in the seuententh conclusion, for if you call the thrée corners of the triangle thræe prickes, and then (as you learned there) if you seeke out the centre to those thræe prickes, and so make it a circle to inclose those thræe prickes in his circumference, you shall perceiue that the same circle shall iustly include the triangle proposed.

¶Example



A.B.C. is the triangle, whose thrée corners I coumpte to bee thræe pointes. Then (as the seuentene conclusion dooeth teache) I seeke a common centre, on whiche I maie make a circle, that shall enclose those thræe prickes that centre. As you see is D, for in D. dooeth the right lines, that passe by the angles of the arche lines, mæte and crosse. And on that centre as you see, haue I made a circle, whiche doeth inclose the thræ angles of the triangle, and consequently the triangle it self, as the conclusion did intende.

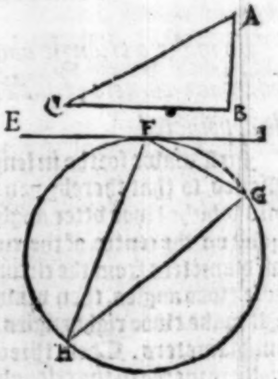
¶The.xxix.conclusion.

To make a triangle in a circle appoincted, whose corners shall bee equall to the corners of any triangle aßigned.

When I will drawe a triangle in a circle appoincted, so that the corners of that triangle, shall bee equall to the corners of any triágle assigned, then must I first drawe a touch line vnto that circle, as the twentie conclusion dooeth teache, and in the verie poincte of the touche, must I make an angle, equall to one angle of the triangle, and that inward toward the circle: Likewise in the same pricke muste I make an other angle, with the other halfe of the touche line, equall to an other corner of the triangle appointed, and then betwene those

those twoo corners, will there resulte a thirde angle, equall to the thirde corner of that triangle. Now where those two lines that entre into the circle, doe touche the circumference (beside the touche line) there sette I twoo prickes, and betwene them I drawe a thirde line. And so haue I made a triangle in a circle appoincted, whose corners bee equall to the corners of the triangle assigned.

¶ Example.



A. B. C. is the triangle appoincted, and F. G. H. is the circle, in which I must make an other Triangle, with like angles, to the angles of A. B, C. the triangle appoincted. Therefore firste I make the touche line D. F. E. And then make I an angle in F. equall too A. whiche is one of the angles of the triangle. And the line that maketh that angle with the touche line, is F. H. whiche I drawe in lengthe untill it touche the edge of the circle. Then againe in the same poincte F. I make an other corner equall to the angle C. and the line that maketh that corner with the touche line, is F. G. whiche also I drawe foorthe untill it touche the edge of the circle. And then haue I made three Angles uppon that one touche line, and in that one poincte F, and those three angles bee equall to the three angles of the triangle assigned, whiche thyng dooeth plainly appeare, in so muche as thei bee equall to twoo right angles, as you maie gesse by the, vj. Theoreme.

And the three angles of euery Triangle, are equall also to two right angles, as the two and twentie Theoreme doeth shewe, so that because thei bee equall to one thirde thyng, thei muste needes bee equall together, as the common sentence saieth. Then doe I drawe a line from G. to H. and that line maketh a triangle F, G, H. whose angles bee equall to the angles of the triangle appoincted. And this triangle is drawen in a circle, as the conclusion did will. The proofe of this conclusion dooeth appeare in the seuentie and fower Theoreme.

¶The.xxx.conclusion.

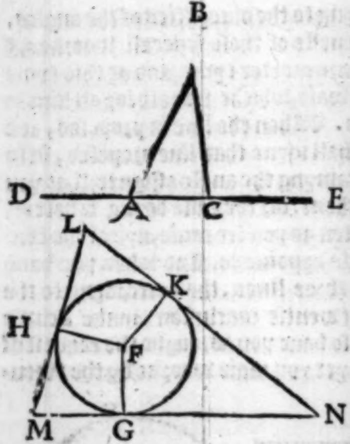
To make a triangle about a circle aßigned, whiche shall haue corners, equall to the corners of any triangle appoincted.

First drawe forthe in length the one side of the triangle assigned, so that thereby you maie haue twoo vtter angles, vnto whiche twoo vtter angles, you shall make twoo other equall on the centre of the circle proposed, drawyng three half diameters from the circumference, whiche shall enclose those twoo angles, then drawe three touche lines, whiche shall make twoo right angles, eche of them with one of those semidiameters. Those three lines will make a triangle, equally cornered to the triangle assigned, and that triangle is drawen about a circle appoincted, as the conclusion did will.

¶Example.

A.B.C. is the triangle assigned, and G.H.K. is the circle appoincted, aboute whiche I must make a triangle, hauyng equall angles to the angles of that triangle A. B. C. Firste therefore I drawe A. C. (whiche is one of the sides of the triangle) in length, that there maie appeare twoo vtter angles in that triangle, as you se B.A.D. and B.C.E.

Then



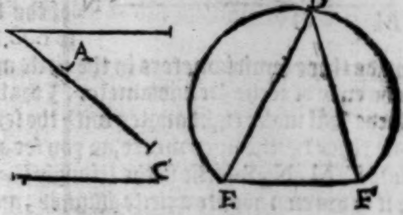
Then drawe I in the circle appoincted a semidiameter, which is here H.F, for F, is the centre of the circle G.H. K. Then make I on that centre an angle equall to the vtter angle B. A. D. and that angle is H. F. K. Likewaies on the same centre by drawyng an other Semidiameter, I make an other angle H. F. G. equall to the seconde vtter angle of the triangle, whiche is B.C.E. And thus haue I made three semidiameters in the circle appoincted. Then at the ende of eache Semidiameter, I drawe a touche line, whiche shal make right angles with the semidiameter. And those three touche lines meete, as you see, and make the triangle L. M. N. whiche is the triangle that I should make, for it is drawen aboute a circle assigned, and hath corners equall to the corners of the triangle appoincted, for the corner M. is equall to C. Likewaies L. to A, and N. to B, whiche thyng you shall better perceiue by the sixte Theoreme, as I will declare in the booke of proofes.

¶The,xxxj.conclusion.

To make a portion of a circle on any right line assigned, whiche shall conteine an angle equall to a right lined angle appoincted.

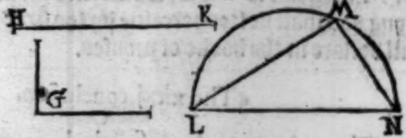
The angle appoincted, maie bee a sharpe angle, a righte angle, either a blunte angle, so that the worke must bee di-

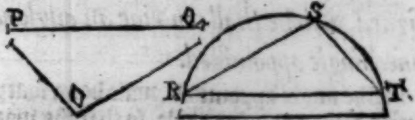
uersely handeled, accordyng to the diuersities of the angles, but consideryng the hardenesse of those seuerall workes, I will omitte them for a more meeter tyme, and at this tyme will shewe you one light waie, whiche serueth for all kindes of angles, and that is this. When the line is proposed, and the angle assigned, you shall ioyne that line proposed, so to the other twoo lines containyng the angle assigned, that you shall make a triangle of them, for the easie doyng whereof, you maie enlarge or shorten as you see cause, nye of the two lines containyng the angle appoincted. And when you haue made a triangle of those three lines, then accordyng to the doctrine of the seuen and twentie conclusion, make a circle about that triangle. And so haue you wrought the request of this conclusion. Whiche yet you maie worke by the twentie and eighte conclusiō also, so that of your line appointed you make one side of the triange be equall to ẏ angle assigned, as your self maie easely gesse.



Example.

Firste for example of a sharpe Angle, lette A. stande and B.C. shall bée the line assigned. Then dooe I make a triãgle, by addyng B.C, as a





third

thirde side to those other twoo, whiche doe include the angle assigned, and that triangle is D. E. F, so that E. F, is the line appoincted, and D. is the angle assigned. Then doe I drawe a portion of a circle about that triangle, from the one ende of that line assigned vnto thother, that is to saie, from E. a long by D. vnto F, whiche portion is euermore greater then the halfe of the circle, by reason that the angle is a sharpe angle. But if the angle be right (as in the seconde example you see it) then shal the portion of the circle that containeth that angle, euermore bee the iuste halfe of a circle. And when the angle is a blunte angle, as in the thirde example doeth propounde, then shall the portion of the circle euermore be lesse then the halfe circle. So in the seconde exãple, G. is the right angle assigned, and H. K. is the line appoincted, and L. M. N. the portion of the circle aunsweryng thereto. In the thirde example, O. is the blunt corner assigned, P. Q. is the line, and R. S. T. is the portion of the circle, that containeth that blunt corner, and is drawen on R. T. the line appoincted.

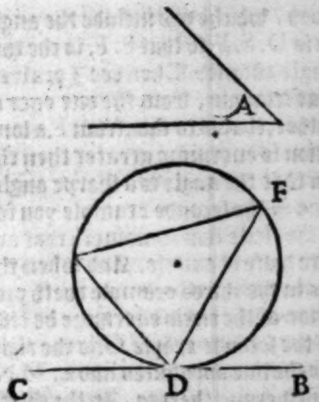
¶The. xxxij. conclusion.

To cutte of from any circle appoincted, a portion cõtaining an angle equall to a right lined angle aßigned

When the angle and the circle are assigned, firste drawe a touche line vnto that circle, and then drawe an other line from the pricke of the touchyng, to one side of the circle, so that thereby those twoo lines dooe make an angle equall to the angle assigned. Then saie I that the portion of the circle of the contrary side to the angle drawen, is the parte that you seke for.

¶Example.

A. is the angle appointed, and D. E. F. is the circle assigned from whiche I must cut awaie a portion that doeth contain an



an angle equall to this angle A. Therefore first I do drawe a touche line to the circle assigned, and that touche line is B, C. the verie pricke of the touche is D. from which D. I drawe a line D:E. so that the angle made of those twoo lines bee equall to the angle appoincted. Then saie I, that the arche of the circle D, F, E, is the arche that I seke after. For if I do deuide that arche in the middle (as here it is doen in F.) and so drawe thence twoo lines, one to A, and the other to E then will the angle F, be equall to the angle assigned.

¶The.xxxiij.conclusion.

To make a square quadrate in a circle aßigned.

Drawe twoo diameters in the circle, so that thei runne a crosse, and that thei make fower right angles. Then drawe fower lines, that maie ioyne the fower endes of those diameters, one to an other, and then haue you made a square quadrate in the circle appoincted.

¶Example.

A, B, C, D. is the circle assigned, and A, C. and B. D. are the twoo diameters, whiche crosse in the centre E. and make fower right corners. Then dooe I make fower other lines, that is A. B, B. C, C. D, and D, A. whiche dooe ioyne together the fower eandes of the twoo diameters, And so is

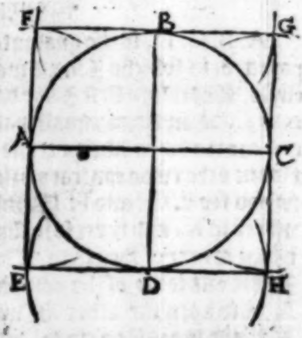
the square quadrate made in the circle assigned, as the conclusion willeth.

¶The,xxxiiiij.conclusion.

To make a square quadrate aboute any circle assigned.

Drawe twoo Diameters in crosse waies, so that thei make fower right angles in the centre. Then with youre Compasse take the lengthe of the halfe diameter, and sette one foote of the Compasse, in eche ende of those diameters, drawing twoo arche lines at euery pitching of the compasse, so shall you haue eight arche lines. Then if you marke the prickes, wherein those arche lines doe crosse, and drawe betwene those fower prickes fower right lines, then haue you made the square quadrate, according to the requeste of the conclusion.

¶Example.



A. B. C. is the circle assigned, in whiche first I drawe twoo Diameters, in crosse waies, makyng fower right angles, and those twoo Diameters are A. C. and B. D. Then sette I my Compasse (whiche is opened, according to the Semidiameter of the saied circle (fixyng one foote in the ende of euery semidiameter, and drawe with other foote twoo arche lines, one on euery side, As firste, when I sette the one foote in A.

 then

then with the other foote I dooe make twoo arche lines, one in E, and an other in F. Then sette I the one foote of the compasse in B. and drawe twoo arche lines F. and G. Likewise settyng the compasse foote in C. I drawe twoo other arche lines, G, and H. and on D. I make twoo other, H, and E. Then from the crossynges of those eighte arche lines, I drawe fower straight lines, that is to saie, E. F. and F. G. also G, H, and H. E, whiche fower straight lines dooe make the square quadrate that I should drawe aboute the circle assigned.

¶The, xxxv. conclusion.

To drawe a circle in any square quadrate appoincted.

First deuide euery side of the quadrate into twoo equall partes, and so drawe twoo lines betwene eche twoo contrary poinctes, and where those twoo lines dooe crosse, there is the centre of the circle. Then sette the one foote of the compasse in that poincte, and stretche forthe the other foote, accordyng to the length of halfe one of those lines, and so make a compasse in the square quadrate assigned.

¶Example.

A. B. C. D, is the quadrate appoincted, in whiche I must make a circle. Therefore first I doe deuide euery side in twoo equall partes, and drawe twoo lines a crosse, betwene eche twoo contrary prickes, as you see E. G. and F. H. whiche meete in K, and therefore shall K, be the centre of the circle. Then do I sette one foote of the compasse in K. and open the other as wide as K. E. and so drawe a circle, whiche is made accordyng to the conclusion.

The

¶The.xxxvj.conclusion.

To drawe a circle aboute a square quadrate.

Drawe twoo lines betwene fower corners of the quadrate, and where thei meete in crosse, there is the centre of the circle that you seeke for. Then set one foote of the compasse in that centre, and extende the other foote unto one corner of the quadrate, and so maie you drawe a circle, whiche shall iustly inclose the quadrate proposed.

¶Example.

A.B.C.D, is the square quadrate proposed, about which I must make a circle. Therefore doe I drawe twoo lines crosse the square quadrate frō angle to angle, as you see A. C. and B.D. And where thei twoo do crosse (that is to saie in E.) there set I the one foote of the compasse, as in the centre, and the other foote I dooe extende unto one angle of the quadrat, as for example to A, and so make a compasse, whiche dooeth iustly inclose the quadrate, accordyng to the mynde of the conclusion.

¶The.xxxvij.conclusion.

To make a twileke triangle, whiche shall haue euery of the twoo angles that lye about the ground line, double to the other corners.

Firste make a circle, and deuide the circumference of it into fiue equall partes. And then drawe from one pricke (whiche you will) twoo lines to twoo other prickes, that is to saie, to the third and fourth pricke, cōptyng that for the first wherehence you drewe bothe those lines. Then drawe the third line to make a triangle with those other twoo, and you haue doen accordyng to the cōclusion, & haue made a twelike

 triangle,

triangle, whose twoo corners aboute the grounde line, are eche of them double to the other corner.

¶Example.

A.B.C.is the circle, whiche I haue deuided into fiue equall portions. And from one of the prickes (whiche is A) I haue drawen twoo lines, A.B, and B.C. whiche are drawen to the thirde and fowerth prickes. Then drawe I the thirde line C.B. whiche is the grounde line, and maketh the triangle, that I would haue, for the angle C. is double to the angle A. and so is the angle B. also.

¶The.xxxviij. conclusion.

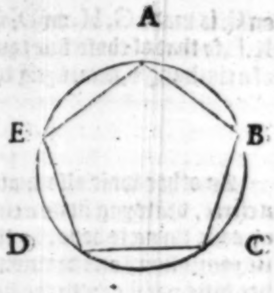
To make a cinckangle of equall sides, and equall corners in any circle appoincted.

Deuide the circle appoincted, into fiue equall partes, as you did in the laste conclusion, and drawe twoo lines from euery pricke to the other twoo that are nexte vnto it. And so shall you make a cinckangle, after the meanyng of the conclusion.

¶Example.

You see here this circle A.B,C,D.E. deuided into fiue equall portions. And from eche pricke twoo lines drawen to the other twoo next prickes, so from A, are drawen twoo sides, one to B, and the other to E; and so from C. to B, and an other

other to D, and likewise of the reste. So that you haue not onely learned hereby, howe to make a sinckangle in any circle, but also how you shall make alike figure spedely, when and where you will, onely drawyng the circle for the intente, readilie to make the other figure (I meane the cinckangle) thereby.



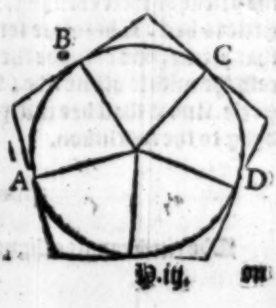
¶The.xxxix.conclusion.

How to make a cinckangle of equall sides and equall angles aboute any circle appoincted.

Deuide firste the circle, as you did in the laste conclusion in the fiue equall portions, and drawe fiue semidiameters in the circle. Then make fiue touche lines, in suche sorte, that euery touche line make twoo right angles, with one of the semidiameters. And those fiue touche lines, will make a cinckangle of equall sides, and equall angles.

¶Example.

A.B.C.D.E. is the circle appoincted, whiche is deuided into fiue equall partes. And vnto euery pricke is drawen a semidiameter, as you see. Then doe I make a touche line in the pricke B. whiche is F,G, making twoo righte Angles with the semidiameter B, and like waies



on C, is made G, H. on D, standeth H, K, and on E, is sette K.L, so that of those fiue touche lines are made the fiue sides of a cinckangle, accordyng to the conclusion.

An otherwaie.

An other waie also maie you drawe a cinckeangle about a circle, drawyng firste a cinckeangle in the circle (whiche is an easie thyng to dooe, by the doctrine of the seuen and thirtie conclusion) and drawyng fiue touche lines, whiche shall bee iuste paralleles to the fiue sides of the cinckeangle in the circle, foreseyng that one of them doe not crosse ouerthwart an other, and then haue you dooen. The example of this (because it is easie) I leaue to your owne exercise.

¶ The.xl.conclusion.

To make a circle in any appoincted cinckeangle of equall sides, and equall corners.

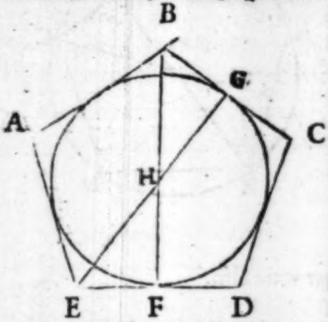
Drawe a plumbe line from any one corner of the cinckeangle, vnto the middle of the side that lieth iust against that angle. And dooe likewaies in drawyng an other line from some other corner, to the middle of the side that lieth against that corner also. And those twoo lines will mete in crosse, in the pricke of their crossyng, shall you iudge the centre of the circle to bee. Therefore sette one foote of the Compasse in that pricke, and extende the other ende of the line, that toucheth the middle of one side, whiche you liste, and so drawe a circle. And it shall bee iustly made in the cinckeangle, accordyng to the conclusion.

¶ Example.

The cinckeangle assigned is A. B. C. D. E, in whiche I

must

must make a circle, wherefore I drawe a right line from the one angle (as frō B.) to the middle of the contrary side (which is E. D.) and that middle pricke is F. Then likewaies from an other corner (as from E.) I drawe a right line to the middle of the side that lieth againste it (whiche is B.C.) and that pricke is G. Now because that these twoo lines dooe crosse in H, I saie that H, is the Centre of the circle whiche I would make. Therefore I set one foote of the compasse in H, and extende the other foote unto G, or F. (whiche are the eandes of the lines that lighte in the middle of the side of that Cinckeangle) and so make I a circle in the cinckeangle, right as the conclusion meaneth.



¶The.xlj.conclusion.

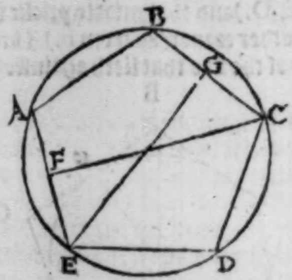
To make a circle aboute any aßigned cinckeangle of equall sides, and equall corners.

Drawe twoo lines within the cinckeangle, from twoo corners to the middle, on the twoo contrary sides (as the last conclusion teacheth) and the poincte of their crossyng shall bee the centre of the circle that I seeke for. Then set I one foote of the compasse in that centre, and the other foote I extende to one of the angles of the cinckeangle, and so drawe a circle about the cinckangle assigned.

¶Example.

A.B.C.D.E, is the cinckangle assigned, aboute whiche I would make a circle. Therfore I drawe first of all two lines (as you se) one from E, to G, and the other frō C, to F. and because

cause thei dooe meete in H. I saie that H. is the centre of the circle that I would haue, wherefore I sette one foote of the Compasse in H. and extende the other to one corner (whiche happeneth firste (for all are like distaunte from H.) and so make I a circle about the cinckangle assigned.



An other waie also.

An other waie maie I dooe it, thus presupposyng any three corners of the cinckangle, to bee three prickes appoincted, vnto whiche I should finde the centre, and then drawyng a circle touchyng them all three, accordyng to the doctrine of the seuentene, one and twentie, and twoo and twētie conclusions. And when I haue founde the centre, then dooe I drawe the circle as the same conclusions doe teache, and this fourtie conclusion also.

¶The.xlij.conclusion.

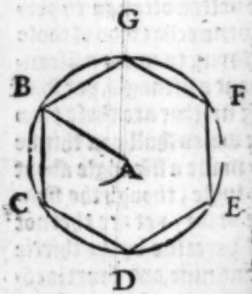
To make a sixeangle of equall sides, and equall angles, in any circle aßigned.

If the centre of the circle bee not knowen, then seeke out the centre, accordyng to the doctrine of the sixteneth conclusion. And with your compasse take the quantitie of the semidiameter iustly. And then sette one foote in one pricke

of

the circumference of the circle, and with the other make a marke in the circumference also towarde bothe sides. Then sette one foote of the compasse stedily in eche of those newe prickes, and poincte out twoo other prickes. And if you haue dooen well, you shall perceiue that there will bee but euen sixe suche deuisions in the circumference, whereby it dooeth well appere, that the side of any siseangle made in a circle, is equall to the semidiameter of the same circle.

¶Example.



The circle is B. C. D. E. F. G, whose Centre I finde to bee A. Therefore I sette one foote of the compasse in A, and doe extende the other foote to B, thereby takyng the semidiameter. Then sette I one foote of the compasse vnremoued in B, and marke with the other foote on eche side C. and G. Then from C. I marke D, and from D, E: from E. mark I F. And thus haue I but one space iuste vnto G. and so haue I made a iuste siseangle of equall sides and equall angles, in a circle appointed.

¶The.xliij.conclusion.

To make a siseangle of equall sides, and equall angles about any circle assigned.

¶The.xliiij.conclusion,

To make a circle in any siseangle appoincted, of equall sides and equall angles.

 The

¶The.xlv.conclusion.

To make a circle about any siseangle, limited of equall sides, and equall angles.

Because you maie easily coniecture the makyng of these figures, by that that is saied before of Cinckeangles, onely consideryng that there is a difference in the number of the sides, I thought beste to leaue these vnto your owne deuice, that you should studie in some thinges, to exercise your wit withall, and that you might haue the better occasion to perceiue, what difference there is betwene eche twoo of those conclusions. For though it seme one thyng to make a siseangle in a circle, and to make a circle about a siseangle, yet shall you perceiue, that it is not one thyng, neither are those twoo conclusions wrought one waie. Like waies shall you thinke of those other twoo conclusions. To make a siseangle about a circle, and to make a circle in a siseangle, though the figures bee one in fashion, when thei are made, yet are thei not one in woorkyng, as you maie well perceiue by the thirtie and seuen, thirtie and eight, thirtie and nine, and fourtie cōclusions, in whiche the same woorkes are taught, touchyng a circle, and a cinckangle: yet this muche will I saie, for your helpe in woorkyng, that when you shall seeke the centre in a siseangle (whether it be to make a circle in it, either about it) you shall drawe the twoo crosse lines, from one angle to the other angle that lieth against it, and not to the middle of any side, as you did in the cinckeangle.

¶The.xlvj.conclusion.

To make a figure of fiftene equall sides and angles in any circle appoincted.

This rule is generall, that howe many sides the figure

shall

ſhall haue, that ſhall bee drawen in any circle, into ſo many partes iuſtely muſte the circle bee deuided. And therefore it is the more eaſier woorke commonly, to drawe a figure in a circle, then to make a circle in an other figure. Now therefore to ende this concluſion, deuide the circle firſte into fiue partes, and then eche of them into three partes againe: Or els firſte deuide it into three partes, and then eche of them into fiue other partes, as you liſte, and can moſte readily. Then drawe lines betwene euery twoo prickes that be nigheſt together, and there will appeare rightly drawen the figure, of fiftene ſides, and Angles equall. And ſo doe with any other figure, of what number of ſides ſo euer it bee.

FINIS.

THE SECOND BOOKE

Of the principles of Geometrie,

containyng certaine Theoremes, whiche maie bee called Approued truthes. And be as it wer the moste certaine groundes, whereon the practike conclusions of Geometrie are founded.

Whereunto are annexed certaine declarations by examples, for the right understandyng of the same, to the eande that the simple Reader, might not iustly complaine of hardnesse or obscuritie, and for the same cause are the demonstrations, and iuste proofes omitted, untill a more conueniente tyme.

If truthe maie trie it ſelf,
By Reaſons prudent ſkill,
If reaſon maie preuaile by right,
And rule the rage of will,
I dare the triall bide,
For truthe that I pretende.
And though ſome liſte at me repine,
Iuſte truthe ſhall me defende.

THE PREFACE vnto the Theoremes.

I Doubte not gentle reader, but as my argument is straunge and vnacquainted with the vulgare tonge so shall I of many men bee straunglie talked of, and as straungly iudged. Some menne will saie peraduenture, I might haue better imploied my tyme in some pleasaunt historie, comprisyng matter of chiualrie. Some other would more haue praised my trauaile, if I had spente the like tyme in some morall matter, either in decisyng some controuersie of Religion. And yet some men (as I iudge) will not mislike this kinde of matter, but then will thei wishe that I had vsed a more certaine order, in placyng bothe the propositions and Theoremes, and also a more exacter proofe of eche of them bothe, by demonstrations Mathematicall. Some also will mislike my shortnesse and simple plainesse, as other of other affections diuersely shall espie somewhat that thei shall thinke blame worthie, and shall misse somewhat, that thei would wishe to haue been here vsed. So that euery manne shall giue his verdicte of me, accordyng to his phantasie, vnto whom ioinctly, I make this my firste aunswere: that as thei are many, and in opinions very diuers, so were it scarse possible to please theim all with any one argument, of what kinde so euer it were. And for my seconde aunswere, I saie thus. That if any one argumente might please them all, then should thei be thankfull vnto me for this kinde of matter. For neither is there any matter more straunge in the Englishe tonge, then this whereof neuer booke was written before now, in that tonge, and therefore ought to delite all theim, that desire to vnderstande

straunge matters, as moste men commonly doe. And againe the practise is so pleasaunte in vsyng, and so profitable in appliyng, that who so euer dooeth delite in any of bothe, ought not of right to mislike this arte. And if any manne shall like the arte well for it self, but shall mislike the fourme that I haue vsed in teachyng of it, to hym I shall saie: First, that I dooe wishe with hym that some other manne, whiche could better haue dooen it, had shewed his good will, and vsed his diligence in suche sorte, that I might haue been thereby occasioned iustely, to haue lefte of my labour, or after my trauaile to haue suppressed my bookes. But sith no manne hath yet attempted the like, as farre as I can learne, I truste all suche as be not exercised in the studie of Geometrie, shall find greate ease and furtheraunce by this simple, plaine, and easie forme of writyng. And shall perceiue the exacte woorkes of Theon, and others that write on Euclide, a greate deale the soner, by this blunt delineacion, afore hande to them taught. For I dare presuppose of theim, that thyng whiche I haue set in my self, and haue marked in others, that is to saie, that it is not easie for a man that shall trauaile in a straũge arte, to vnderstande at the beginnyng, bothe the thyng that is taught, and also the iust reason why it is so. And by experiẽce of teaching, I haue tried it to be true, for whẽ I haue taught the proposition, as it imported in meanyng, and annexed the demonstration withall, I did perceiue that it was greate trouble, and painfull vexatiõ of mynde to the learner, to comprehende bothe those thynges at ones. And therefore did I proue firste to make theim to vnderstande the sence of the propositions, and then afterwarde did thei conceiue the demonstrations muche soner, whẽ thei had the sentence of the propositions firste ingrafted in their myndes. This thyng caused me in bothe these bookes to omitte the demonstrations, and to vse onely a plaine forme of declaration, whiche might beste serue for the firste introduction. Whiche example hath been vsed by other learned men before now, for not onely Georgius Ioachimus Rheticus, but also Boetius that

wittie clarke, did set forthe some whole bookes of Euclide, without any demonstration, or any other declaration at all. But and if I shall hereafter perceiue that it maie be a thākfull trauaile, to set forth the propositions of Geometrie with demonstrations, I will not refuse to dooe it, and that with sondrie varieties of demonstrations, bothe pleasaunte and profitable also. And then will I in like maner prepare to set forthe the other bookes, whiche now are lefte vnprinted, by occasiō not so muche of the charges in cutting of the figures, as for other iuste hinderaunces, whiche I truste hereafter shall be remedied. In the meane season if any manne muse, why I haue set the Conclusiōs before the Theoremes, seyng many of the Theoremes seme to include the cause of some of the conclusions, and therefore ought to haue gone before thē, as the cause goeth before the effecte. Here vnto I saie, that although the cause dooe goe before the effecte in order of nature, yet in order of teachyng, the effecte must bée first declared, and then the cause thereof shewed, for so shall men best vnderstande thynges. First to learne that suche thinges are to be wrought, and secondarily what thei are, and what thei doe import, and then thirdly, what is the cause therof. An other cause why that the Theoremes be put after the conclusions is this, when I wrote these firste conclusions (whiche was fower yeres passed) I thought not then to haue added any Theoremes, but next vnto the conclusions, to haue taught the order how to haue applied theim to worke, for drawyng of plattes, and suche like vses. But afterwarde consideryng the greate commoditie that thei serue for, and the light that thei dooe geue to all sortes of practise Geometricall, beside other more notable benefites, whiche shall bée declared more specially in a place conueniente, I thought beste to geue you some tast of them, and the pleasaunt contemplation of suche Geometricall propositions, whiche might serue diuersly in other bookes for the demonstrations, and proofes of all Geometricall workes. And in them, as well as in the propositiōs I haue drawen in the Linearie examples many times more

lines, then be spoken of in the explication of them, whiche is doen to this intent, that if any man list to learne the demonstrations by harte, as some learned men haue iudgee best to doe) those same men should finde the Linearie examples to serue for this purpose, and to want no thyng needefull to the iuste proofe, whereby this booke maie bee well approued, to be more complete then many men would suppose it.

And thus for this tyme I will make an ende, without any larger declaration of the commodities of this art, or any farther answeryng to that maie bee obiected against my handelyng of it, willyng theim that mislike it, not to meddle with it: and vnto those that will not disdain the studie of it, I promise all suche aide as I shalbe able to shewe for their farther proceadyng, bothe of the same, and in all other commodities that therof maie ensue. And for their incouragement I haue here annexed the names and brief argumentes of suche bookes, as I intende (God willyng) shortly to set forth, if I shall perceiue that my paines maie profite other, as my desire is.

¶ *The brief argumentes of suche bookes as are appoincted shortly to bee sette forthe by the aucthour hereof.*

The seconde parte of Arithmetike, teachyng the workyng by fractions, with extraction of rootes, bothe square and cubike: and declaryng the rule of allegation, with sundrie pleasaunt examples in metalles and other thynges. Also the rule of false position, with diuers examples not onely vulgar, but some appertainyng to the rule of Algeber, applied vnto quantities, partly rationall, and partly surde.

The art of Measuryng by the quadrate Geometricall, and the disorders cōmitted in vsyng the same, not onely reueled but reformed also (as muche as to thinstrument pertaineth) by the deuise of a newe quadrate, newely inuented by the aucthour hereof.

The arte of measuryng by the Astronomers staffe, and by the Astronomers ryng, and the forme of making them both.

The arte of makyng of Dials, bothe for the daie and the nighte, with certaine newe formes of fixed Dialles for the

Mone, and other for the sterres, whiche maie be set in glasse windowes, to serue by daie & by night. And how you maie by those Dials knowe in what degree of the Zodiake, not onely the Sonne, but also the Mone is. And how many howers old she is. And also by the same Diall to knowe whether any eclipse shalbe that moneth, of the Sonne, or of the Moone.

The makyng and vse of an Instrumente, whereby you maie not onely measure the distaunce at once, of all places that you can see together, how muche eche one is from you, and euery one from other, but also thereby to drawe the plot of any countrie that you shall come in, as iustely as maie be, by mannes diligence and labour.

The vse bothe of the Globe and the Sphere, and therein also of the art of Nauigation, and what instrumentes serue beste thereunto, and of the true latitude and longitude of regions and townes.

Euclides woorkes in fower partes, with diuers demonstrations Arithmeticall and Geometricall, or Linearie. The firste parte of platte fourmes. The seconde of numbers and quantities surde, and irrationall. The thirde of bodies and solide formes. The fowerth of perspectiue, and other thynges thereto annexed.

Beside these I haue other sundrie workes, partly ended, and partly to be ended. Of the peregrination of man, and the originall of all Nations: The state of tymes, and mutations of realmes: The Image of a perfecte common wealth, with diuers other woorkes in naturall sciences: Of the wonderfull woorkes and effectes in beastes, plantes, and mineralls, of whiche at this tyme, I will omitte the argumentes, because thei doe appertaine little to this arte, and handle other matters in an other sorte.

To haue, or leaue,
Now maie you chuse.
No paine to please,
Will I refuse.

The Preface.

[illegible]

THE PREFACE.

declaryng briefly the commodities of *Geometrie*, and the neceſſitie thereof.

GEOMETRIE maie thinke it ſelf to ſuſtaine greate iniurie, if it ſhal bee inforced either to ſhewe her manifolde commodities, or els not to preaſe into the ſight of menne, and therefore mighte this waies anſwere briefly: Either I am able to dooe you muche good, or els but little. If I bee able to dooe you muche good, then bee you not your owne friendes, but greatly your owne enemies, to make ſo little of me, whiche maie profite you ſo muche. For if I wer as vncurteous, as you vnkinde, I ſhould vtterly refuſe to do them any good, whiche will ſo curiouſly put me to the triall and proofe of my commodities, or els to ſuffer exile, and namely ſith I ſhall onely yelde benefites to other, and receiue none againe. But and if you could ſaie truely, that my benefites bee neither many, nor yet greate, yet if thei bee any, I doe yelde more to you, then I doe receiue againe of you, and therefore I ought not to be repelled of them that loue them ſelf, although thei loue me not at all for my ſelf. But as I am in nature a liberall ſcience, ſo can I not againſte nature contende with your inhumanitie, but muſte ſhewe my ſelf liberall euen to myne enemies. Yet this is my comfort againe, that I haue none enemies, but them that know me not, and therefore maie hurte them ſelues, but can not noye me. If thei diſpraiſe the thyng that thei knowe not, all wiſe menne will blame them, and not credite them. And if thei thinke thei knowe me, let them ſhewe one vntruthe and errour in me, and I will giue the victorie.

Yet can no humaine Science saie thus, but I onely, that there is no sparke of vntruthe in me: but all my doctrine and workes are without any blemishe of errour, that mannes reason can discerne. And nexte vnto me in certaintie are my three sisters, Arithmetike, Musike, and Astronomie, whiche are also so nere knitte in amitie, that he that loueth the one, can not despise the other, and in especiall Geometrie, of whiche not onely these three, but all other artes doe borowe greate aide, as partly hereafter shall be shewed. But first I will beginne with the vnlearned sorte, that you maie perceiue how that no arte cā stand without me. For if I should declare how many waies my helpe is vsed, in measuryng of ground, for medowe, corne, and wood: in hedging, in dichyng and in stakes makyng, I thinke the poore Husbande manne would bee more thankefull vnto me, then he is now, whiles he thinketh that he hath small benefite by me. Yet this may be coniecture certainly, that if he kepe not the rules of Geometrie, he can not measure any grounde truely. And his dichyng, if he kepe not a proportion of bredth in the mouthe, to the bredth of the bottome, and iuste slopenesse in the sides, agreable to them bothe, the diche shalbe faultie many waies. When he doeth make stackes for corne, or for heye, he practiseth good Geometrie, els would thei not long stand: so that in some stackes, whiche stande on fower pillers, and yet made rounde, doe increase greater and greater a good heighte, and then again turne smaller and smaller vnto the top: you may se so good Geometrie, that it were verie difficulte to counterfaite the like in any kynde of buildyng. As for other infinite waies that he vseth my benefite, I ouerpasse for shortnesse.

Carpenters, Karuers, Joyners, and Masons, doee willingly acknowledge, that thei can worke nothyng without reason of Geometrie, in so muche that thei chalenge me as a peculiare science for thē. But in that thei should doe wrong to all other men, seyng euery kinde of men haue some benefite by me, not onely in buildyng, whiche is but other mens costes, and the arte of Carpenters, Masons, and other aforesaied,

faied, but in their owne priuate profession, wherof to auoide tediousnesse I make this rehearsall.

Sith Merchauntes by Shippes greate riches doe winne,
 I maie with good right at their feete beginne.
The Shippes on the sea with Saile and with Ore,
 VVere first founde, and still made, by *Geometries* lore.
Their Compas, their Carde, their Pulleis, their Ankers,
 were founde by the skill of wittie *Geometers*.
To sette forthe the Capstocke, and eche other parte,
 would make a greate showe of *Geometries* arte.
Carpenters, Karuers, Ioyners and Masons,
 Painters and Limmers with suche occupations,
Broderers, Goldsmithes, if thei bee cunnyng,
 Must yelde to *Geometrie* thankes for their learnyng.
The Carte and the Plowe, who doeth them well marke,
 Are made by good *Geometrie*. And so in the warke
Of Tailers and Shoomakers, in all shapes and fashion,
 The woorke is not praised, if it wante proportion.
So weauers by *Geometrie* had their foundation,
 Their Loome is a frame of straunge imagination.
The wheele that doeth spinne, the stone that doeth grinde,
 The Mill that is driuen by water or winde,
Are woorkes of *Geometrie* straunge in their trade,
 Fewe could them deuise, if thei were vnmade.
And all that is wrought by waight or by measure,
 without proofe of *Geometrie* can neuer be sure,
Clockes that be made the tymes to deuide,
 The wittiest inuention that euer was spied,
Now that thei are common thei are not regarded.
 The artes man contemned, the woorke vnrewarded.
But if thei were scarse, and one for a showe,
 Made by *Geometrie*, then should men knowe,
That neuer was arte so wonderfull wittie,
 So needefull to man, as is good *Geometrie*.
The first findyng out of euery good arte,
 Seemed then vnto men so godlie a parte.

That no recompence might satisfie the finder,
But to make hym a God, and honour hym for euer,
So *Ceres* and *Pallas*, and *Mercurie* also,
Eolus and *Neptune*, and many other mo,
VVere honoured as goddes, because thei did teache,
First tillage and weauyng, and eloquent speache,
Or windes to obserue, the seas to saile ouer,
Thei were called goddes for their good indeuour.
Then were men more thankefull in that golden age:
This yron worlde now vngratefull in rage,
VVill yelde thee thy reward for trauaile and paine,
VVith sclaunderous reproche, and spitefull disdaine.
Yet though other men vnthankfull will be,
Suruayers haue cause to make muche of me.
And so haue all Lordes, that landes doe possesse:
But Tenauntes I feare will like me the lesse.
Yet doe I no wrong but measure all truely,
And yelde the full right to euery man iustely.
Proportion *Geometricall* hath no man opprest,
If any bee wronged, I wishe it redrest.

But now to procede with learned profession, in Logike and Rhetorike, and all partes of Philosophie, there needeth none other proofe then Aristotle his testimonie, whiche without Geometrie proueth almoste nothyng. In Logike all his good syllogismes and demonstrations, be declared by the principles of Geometrie. In Philosophie, neither motion, nor tyme, nor ayrie impressions, could he aptly declare, but by the helpe of Geometrie, as his workes doe witnesse. Yea the faculties of the mynde doeth he expresse by similitude, to figures of Geometrie. And in moral Philosophie he thought that Justice could not be well taught, nor yet well executed without proportion Geometricall. And this estimation of Geometrie he maie seme to haue learned of his maister Plato, whiche without Geometrie would teache nothyng, neither admitte any to heare hym, excepte he were experte in Geometrie. And what meruail if he so muche estemed Geometrie,

metrie, seyng his opinion was, that God was alwaies woorkyng by Geometrie: Whiche sentence Plutarche declareth at large. And although Plato doe vse the helpe of Geometrie in all the moste waightie matter of a common wealthe, yet it is so generall in vse, that no small thynges can bee well doen without it. And therefore saieth he: that Geometrie is to be learned, if it wer for none other cause, but that al other artes are bothe soner & more surely vnderstand by help of it.

What greate helpe it dooeth in Phisike, Galen doeth so often and so copiously declare, that no man whiche hath red any booke almoste of his, can bee ignoraunte thereof. In so muche that he could neuer cure well a round vlcere, till reason Geometricall did teache it hym. Hippocrates is earnest in admonishyng that studie of Geometrie, must prepare the waie to Phisike, as well as to all other artes.

I should seme somwhat to tedious, if I should recken vp, how the diuines also in their misteries of scripture, doe vse helpe of Geometrie: and also that lawyers can neuer vnderstande the whole lawe, no nor yet the firste title thereof exactly without Geometrie. For if Lawes can not well bee established, nor iustice duely executed without Geometrical proportion, as bothe Plato is his Politike bookes, and Aristotle in his Moralles doe largely declare. Yea sith Lycurgus that chief lawmaker emongest the Lacedemonians, is most praised, for that he did chaunge the state of their Common wealthe, from the proportion Arithmeticall, to a proportion Geometricall, whiche without knowledge of bothe he could not do, then is it easie to perceiue how necessarie Geometrie is for the lawe, and studentes thereof. And if I shall saie precisely, and frely as I thinke, he is vtterly destitute of all abilitie to iudge any arte, that is not somewhat experte in the Theoremes of Geometrie. And that caused Galene to saie of hym self, that he could neuer perceiue what a demonstratiō was, no not so muche, as whether there were any or none, till he had by Geometrie gotten abilitie to vnderstande it, although he heard the beste teachers that were in his tyme.

It should be to long and nedelesse also to declare, what helpe all other artes Mathematicall haue by Geometrie, sith it is the ground of all their certaintie, and no man studious in thē is so doubtfull thereof, that he shall neede any persuasion to procure credite thereto. For he can not read.ij. lines almoste in any Mathematicall science, but he shall espie the nedefulnes of Geometrie. But to auoide tediousnes I wil make an ende hereof with that famous sentēce of auncient Pythagoras, *That who so will trauaile by learnyng to attaine wisedome, shall neuer approche to any excellencie without the artes* Mathematicall *and especially* Arithmetike *and* Geometrie.

And if I shall somewhat speake of noble men, and gouernours of realmes, how needefull Geometrie maie bee vnto them, then must I repete all that I haue saied before, sith in them ought all knowledge to abound, namely that maie appertaine either to good gouernaunce in tyme of peace, either wittie pollicies in tyme of warre. For ministration of good lawes in tyme of peace Lycurgus example, with the testimonies of Plato & Aristotle maie suffice. And as for warres, I might thinke it sufficiente that Vegetius hath written, and after hym Valturius in commendation of Geometrie, for vse of warres, but all their woordes seme to saie nothyng, in comparison to the example of Archimedes worthy woorkes made by Geometrie, for the defence of his Countrey, to reade the wonderfull praise of his wittie deuises, set foorthe by the moste famous histories of Liuius, Plutarche, and Plinie, and all other historiographiers, whiche write of the strong siege of Syracusæ, made by that valiaunte Capitaine, and noble warriour Marcellus, whose power was so greate, that all menne meruailed how that one Citie, could withstande his wonderfull force so longe. But muche more would thei meruaile, if thei vnderstoode that one man onely did withstande all Marcellus strength, and with counter engines destroied his engines, to the vtter astonishement of Marcellus, and all that were with hym. He had inuented suche balastelas that did shoote out a hundred dartes at one

shoote,

shoote , to the greate destruction of Marcellus Souldiours, wh reby a fonde tale was spred abrode, how that in Syracusæ there was a wonderfull Gyante, whiche had an hundred handes, and could shote a hūdred dartes at once. And as this fable was spred of Achimedes, so many other haue been faïned to be gyantes and monsters, because thei did suche thinges whiche farre passed the witte of the common people. So did thei feigne Argus to haue an hundred eyes, because thei heard of his wonderfull circumspection, and thought that as it was aboue their capacitie, so it could not be, unlesse he had a hundred eyes. So imagined thei Ianus to haue twoo faces, one loking forward, and an other backward, because he could so wittily compare thynges past, with thynges that were to come, and so duely ponder them, as if thei were all presente. Of like reasō did thei fein Lynceus to haue suche sharp sight that he could se through walles and hilles, because peraduenture he did by naturall iudgement, declare what cōmodities might be digged out of the grounde. And an infinite nomber like fables are there, whiche sprange all of like reason.

For what other thyng meaneth the fable of the greate gyante Atlas, whiche was imagined to beare up heauen on his shulders? but that he was a man of so high a witte, that it reached unto the skie, and was so skilfull in Astronomie, and could tell before hande of Eclipses, and other like thynges, as truely as though he did rule the sterres, and gouerne the Planettes.

So was Eolus accompted God of the windes, and to haue thē all in a caue at his pleasure, by reason that he was a wittie man in naturall knowledge, ⁊ obserued well the chaunge of weathers, and was the first that taught the obseruatiō of the windes. And like reason is to be giuē of all the old fables.

But to retourne againe to Archimedes, he did also by art perspectiue (whiche is a part of Geometrie) deuise suche glasses within the towne of Syracusæ, that did bourne their enemies shippes a greate waie from the towne, whiche was a meruailous politike thynge. And if I should repeate the

varietie,

varieties of suche straunge inuentions, as Archimedes and others haue wrought by Geometrie, I should not onely excede the order of a Preface, but I should also speake of suche thynges as can not well bee understande in talke, without some knowledge in the principles of Geometrie.

But this will I promise, that if I maie perceiue my paines to bee thankfully taken, I will not onely write of suche pleasaunte inuentions, declaryng what thei were, but also will teache how a greate nomber of them were wroughte, that thei maie be practised in this tyme also. Whereby shal be plainly perceiued, that many thynges seme impossible to bee doen, whiche by arte maie verie well bee wrought. And when thei bee wrought, and the reason thereof not understande, then saie the vulgare people, that those thynges are dooen by Negromancie. And hereof came it that Frier Bakon was accompted so greate a Negromancier, whiche neuer used that arte (by any coniecture that I can finde) but was in Geometrie, and other Mathematicall sciences so experte, that he could doe by them suche thynges, as were wonderfull in the sight of moste people.

Great talke there is of a glasse that he made in Oxforde, in whiche men might se thinges that wer doen in other places, and that was iudged to bee doen by power of euill spirites. But I knowe the reason of it to bee good and naturall, and to be wrought by Geometrie (sith perspectiue is a parte of it) and to stande as well with reason, as to see your face in common glasse. But this conclusion and other diuers of like sort, are more meete for Princes, for sundrie causes, then for other men, and ought not to be taught commonly. Yet to repete it, I thought good for this cause, that the worthines of Geometrie might the better be knowen, & partly understanding giuen, what wonderfull thinges maie be wrought by it and so consequently how pleasant it is, & how necessary also.

And thus for this tyme I make an ende. The reason of some thynges dooen in this booke, or omitted in the same, you shall finde in the Preface before the Theoremes.

The Theoremes of Geometrie,

before vvhiche are sette forthe certain grauntable requestes, whiche serue for demonstrations *Mathematicall.*

That from any pricke to one other, there maie be drawen a right line.

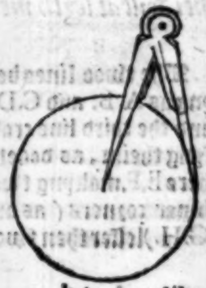
As for example A.————B. A.beyng one pricke, and B. the other, you maie drawe betwene them, from the one to the other, that is to saie, from A. vnto B.and from B.to A.

That any right line of measurable length, maie bee drawen forthe longer, and straight.

Example of A.B, whiche as it is a line of measurable length, so maie it be drawen forthe farther, as for example vnto C.and that in true straightnesse without crokyng.

A B C

That vpon any centre, there maie bee made a circle of any quantitie that a man will:



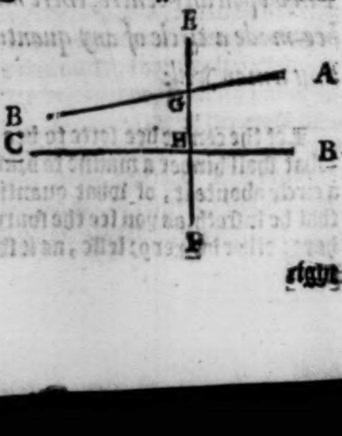
Let the centre bee sette to bee A. what shall hinder a manne to drawe a circle aboute it, of what quantitie that he lusteth, as you see the fourme here: ether bigger or lesse, as it shall

like hym to doe.

That all right angles bee equall eche to other.

Set for an example A. and B. of which twoo though A. ſeme the greater angle, to ſome men of ſmall experience, it happeneth onely becauſe that the lines aboute A. are longer then the lines about B. as you maie proue by drawyng them longer, for ſo ſhall B. ſeeme the greater angle, if you make his lines longer, then the lines that make the angle A. And to proue it by demonſtration, I ſaie thus. If any twoo right corners bee not equall, then one right corner is greater then an other, but that corner whiche is greater then a right angle, is a blunt corner (by his definition) ſo muſt one corner bee bothe a right corner, and a blunt corner alſo, whiche is not poſſible. And againe: the leſſer right corner muſt bee a ſharpe corner, by his definition, becauſe it is leſſe then a right angle, whiche thyng is impoſſible. Therefore I conclude, that all right angles bee equall.

If one right line doe croſſe twoo other right lines, and make twoo inner corners of one ſide leſſer then two right corners, it is certain, that if thoſe ij. lines be drawē forth right on that ſide that the ſharpe inner corners be thei will at lēgth mete together, and croſſe one an other.



The twoo lines beyng as A.B. and C.D, and the third line croſſyng theim, as dooeth here E.F. makyng twoo inner corners (as are G.H.) leſſer then twoo

right corners, sith eche of them is lesse then a right corner, as your eyes maie iudge, then saie I, if those twoo lines A, B. and C, D. bee drawen in length on that side that G. and H. are, thei will at length meete, and crosse one an other.

Twoo right lines make no platte fourme.

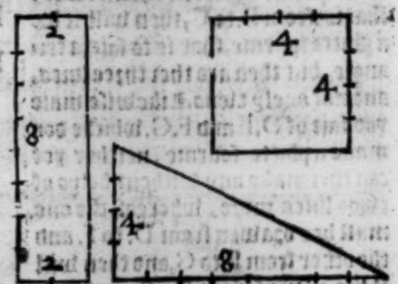
A platte fourme, as you heard before, hath bothe length and breadth, and is inclosed with lines, as with his boundes, but twoo right lines cannot inclose all the bondes of any platte fourme. Take for an example, firste these twoo right lines A, B. and A. C. whiche meete together in A. but yet cannot bee called a platte fourme, because there is no bonde from B. to C. but if you will drawe a line betwene theim twoo, that is, from B. to C, then will it bee a platte fourme, that is to saie, a triangle, but then are thei three lines, and not onely twoo. Likewise maie you saie of D.E. and F.G. whiche doe make a platte fourme, neither yet can thei make any without helpe of twoo lines more, whereof the one must bee drawen from D. to F. and the other from E. to G, and then will it bee a long square. So then of twoo right lines can bee made no platte fourme. But of twoo crooked lines bee made a platte fourme, as you se in the eye fourme. And also of one right line, and one croked line, maie a platte fourme bee made, as the Semicircle F. doeth sette forthe.

Certaine common sentences manifest to sence, and acknowledged of all menne.

¶The first common sentence.

WHat so euer thynges bee equall to one other thyng, those same bee equall betwene them selues.

Examples thereof you maie take bothe in greatnes, and also in noumber. Firste (though it pertaine not properly to Geometrie, but to helpe the understandyng of the rules, whiche maie bee wroughte by bothe Artes) thus maie you perceiue. If the somme of money in my purse, and the money in your purse bee equal eche of them, to the money that any other manne hath, then muste needes your money and myne be equall together. Likewise, if any twoo quantities, as A. and B, bee equall to an other as unto C, then muste needes A. and B. bee equall eche to other, as A. equall to B. and B. equall to A, whiche thyng the better to perceiue, tourne these quantities into nomber, so shall A, and B. make sixtene, and C. as many. As you maie perceiue by multiplyng the number of their sides together.



¶The seconde common sentence.

And if you adde equll portions to thynges that be equall, what ſo amounteth of them ſhall be equall.

Example. If you and I haue like ſommes of money, and then receiue eche of vs like ſommes more, then our ſommes will bee like ſtill. Alſo if A. and B. (as in the former example) bee equall, then by addyng an equall portion to them bothe, as to eche of them, the quarter of A. (that is fower) thei will bee equall ſtill.

¶ The thirde common ſentence.

And if you abate euen portions from thynges that are equall, thoſe partes that remaine ſhall bee equall alſo.

This you maie perceiue by the laſte example. For that that was added there, is ſubſtracted here. And ſo thone doeth approue the other.

¶ The fowerth common ſentence.

If you abate equalle partes from vnequall thynges, the remainers ſhall bee vnequall.

As becauſe that a hundreth and eight and fourtie bee vnequall, if I take tenne from them bothe, there will remaine ninetie and eight and thirtie, whiche are alſo vnequall. And likewiſe in quantities it is to bee iudged.

¶ The fifte common ſentence.

VVhen euen portions are added to vnequall thynges,

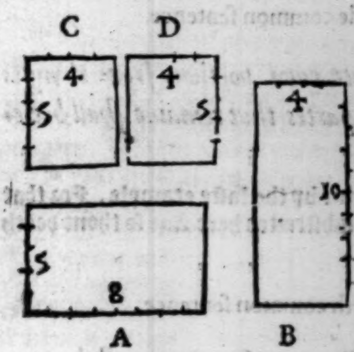
 ges,

ges, those that amounte shall bee vnequall.

So if you adde twentie to fiftie, and likewaies to ninetie, you shall make seuentie, and a hundred and tenne, which are no lesse vnequall, then were fiftie and ninetie.

¶The sixte common sentence.

If twoo thynges bee double to any other, those same twoo thynges are equall together.



Because A, and B, are eche of theim double to C, therfore must A, and B, nedes be equall together. For as fiue tymes eight maketh fowertie which is double to foure tymes fiue, that is xx. so fower times tenne, likewise is double to xx. (for it maketh fowertie) and therefore must nedes be equall to fowertie.

¶The seuenth common sentence.

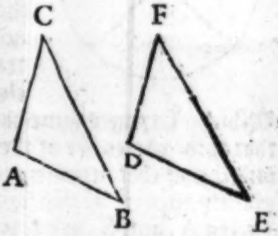
If any twoo thynges bee the halfes of one the other thyng, then are thei twoo equall together.

So are D. and C. in the laste example equall together, because thei are eche of them the halfe of A, either of B, as their number declareth.

¶The eight common sentence.

If

If any one quantie bee laied on an other, and thei agree, ſo that the one exceadeth not the other, then are thei equall together.



As if this figure A.B.C. bée laied on that other D.E.F. ſo that A. bee laied to D. B. to E. and C. to F. you ſhall ſee theim agree in ſides exactly, and the one not to excede the other, for the line A.B. is equall to D.E. and the third line C.A. is equal to F.D. ſo that euery ſide in the one is equall to ſome one ſide of the other, wherefore it is plaine, that the twoo triangles are equall together.

¶The nineth common ſentence.

Euery whole thyng is greater then any of his partes.

This ſentence needeth none example. For the thyng is more plainer then any declaration, yet conſideryng that other common ſentence that foloweth nexte that.

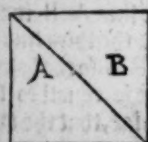
¶The tenth common ſentence.

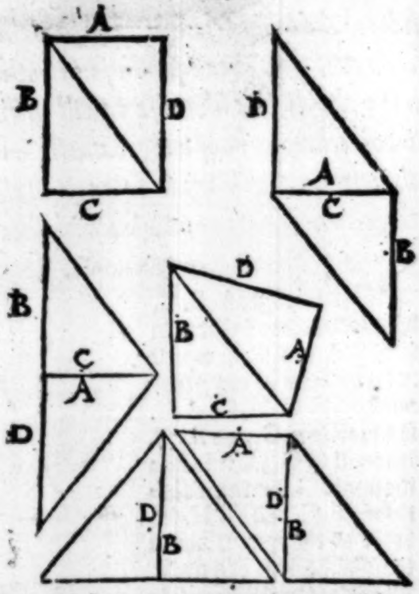
Euery whole thyng is equall to all his partes taken together.

It ſhall bee meete to expreſſe bothe with one example, for this laſte ſentence many menne at the firſt hearyng doe make a doubte. Therefore, as in this example of the circle deuided into ſundrie partes it doeth appere, that no part can bee ſo greate as the whole circle, (accordyng to the meanyng of the eight ſentence) ſo yet it is certaine, that all thoſe eight

partes

partes together bee equall vnto the whole circle. And this is the meanyng of that common Sentence, (whiche many vse, and fewe dooe rightly vnderstande) that is to saie, that All the partes of any thyng are nothyng els, but the whole. And cōtrary waies: The whole is nothyng els, but all his partes taken together. Whiche saiynges some haue vnderstande to meane thus: that al the partes are of thesame kinde that the whole thing is: but that that meanyng is false, it dooeth plainly appeare by this figure A.B. whose partes A. and B. are triangles, and the whole figure is a square, and so are thei not of one kynde. But and if thei applie it to the matter or substaunce of thynges (as some dooe) then is it moste false, for euery compounde thing is made of partes of diuerse matter and substaunce. Take for example a manne, a house, a booke, and all other compounde thynges. Some vnderstande it thus, that the partes all together, can make none other fourme, but that that the whole dooeth shewe, whiche is also false, for I maie make fiue hundred diuerse figures, of the partes of some one figure, as you shall better perceiue in the thirde booke. And in the meane season take for an example this square figure folowyng A.B.C.D, whiche is deuided but into twoo partes, and yet (as you see) I haue made fiue figures more beside the firste, with onely diuerse ioynyng of those twoo partes. But of this shall I speake more largely in an other place, in the meane season, contente youre self with these principles, whiche are certaine of the chief groundes, whereon all demonstrations Mathematicall are fourmed, of whiche though the moste parte seme so plaine, that no childe dooeth doubt of them, thinke not therefore that the Arte vnto whiche thei serue, is simple, either childishe, but rather consider, how certaine





faine the prooses of that artes is, that hath for his r undes suche plain truthes, and as I maie saie, such undoubtefull & sensible principles. And this is ẏ cause why all learned mẽ doeth approue the certainetie of Geometrie, and cõsequently of the other Artes Mathematicall, whiche haue the groundes (as Arithmetike, Musicke, and Astronomie) aboue all other artes and sciences, that bee used emongest men. Thus muche haue I saied of the first principles, and now wil I goe on with the Theoremes, whiche I doe onely by examples declare, mindyng to reserue the proofes to a peculiare booke, whiche I will then sette forthe, when I perceiue this to bee thankfully taken of the readers of it.

The Theoremes of Geometrie, briefly declared by shorte examples.

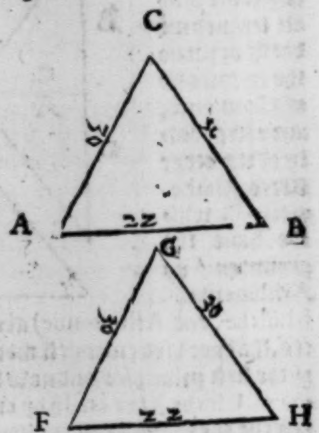
¶The first Theoreme.

WHen twoo triangles bee so drawen, that the one of thiem hath twoo sides equall to twoo

 sides

sides of the other triangle, and that the angles enclosed with those sides, bee equall also in bothe triangles, then is the thirde side likewise equall in theim. And the whole triangles be of one greatnesse, and euery angle in the one equall to his matche angle in the other, I meane those angles that bee inclosed with like sides.

¶Example.



This triangle A. B. C. hath twoo sides (that is to saie) C. A. and C. B. equall to twoo sides of the other triangle F. G. H, for A. C. is equall to F. G. and B. C. is equall to G. H. And also the angle G. contained betwene F. G. and G. H. for bothe of them aunswer to the eight parte of a circle. Therefore doeth it remain that A. B. whiche in the thirde line in the firste triangle, doeth agre in length with F. H. whiche is the thirde line in the seconde triangle, and the whole triangle A. B. C. muste needes bee equal to the whole triangle F, G, H. And euery corner equall to his matche, that is to saie, A. equall to F. B. to H. and C. to G. for those bee called matche corners, whiche are inclosed with like sides, either els doe lye against like sides.

¶The seconde Theoreme.

In twileke triangles the twoo corners that bee about

boute the grounde line, are equall together. And if the sides that bee equall, be drawen out in length, then will the corners that are vnder the grounde line, bee equall also together.

¶Example.

A.B.C. is a twiseke triangle, for the one side A. C, is equall to thother side B.C. And therefore I saie that the inner corners A. and B, which are about the grounde lines, (that is A.B.) be equall together. And farther if C. A. and C. B. bee drawen forthe vnto D. and E. as you see that I haue drawen them, then saie I that the twoo vtter angles vnder A. and B. are equall also together: as the Theoreme saied. The proofe whereof, as of all the rest, shall appeare in Euclide, whom I intende to sette forthe in Englishe, with sundrie newe additions, if I maie perceiue that it will be thankfully taken.

¶The thirde Theoreme.

If in any triangle there bee twoo angles equall together, then shall the sides, that lye against those angles be equall also.

¶Example.

This triangle A.B.C. hath twoo corners equall eche to other, that is A. and B. as I doe by supposition limite, wherefore it foloweth that the side A. C, is equall to that other side B.C, for the side A. C, lieth againste the angle B, and the side B.C. lieth against the angle A.

¶The.iiij.Theoreme.

When twoo lines are drawen from the endes of any one line, and meete in any poincte, it is not possible to drawe twoo other lines of like lengthe eche to his matche, that shall beginne at the same poinctes, and ende in any other poincte then the twoo first did.

¶Example.

The first line is A.B. on whiche I haue erected twoo other lines A.C, and B.C. that mete in the pricke C. wherefore I saie, it is not possible to drawe twoo other lines from A. and B. whiche shall meete in one poincte (as you see A.D. and B.D. meete in D.) but that the matche lines shall bee vnequall, I meane by matche lines, the twoo lines on one side, that is the twoo on the right hande, or the twoo on the left hande, for as you see in this example A.D. is longer then A.C. and B.C. is longer then B.D. And it is not possible, that A.C. and A.D. shall bee of one length, if B.D. and B.C. bee like long. For if one couple of matche lines bee equall (as the same example A.E. is equall to A.C. in lengthe) then muste B.E. nedes bee vnequall to B.C. as you see, it is here shorter.

¶The.v.Theoreme.

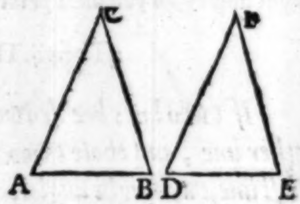
If twoo triangles haue their twoo sides equall one to an other, and their grounde lines equall also, then

shall

shall their corners, whiche are contained betwene like sides, bee equall one to the other.

¶Example.

Because these twoo triangles A.B.C. and D.E.F. haue twoo sides equall one to an other. For A.C. is equall to D.F. and B.C. is equall to E.F. and againe the grounde lines A.B. and D.E. are like in length, therefore is eche angle of the one triangle, equall to eche angle of the other, comparyng together those angles, that are contained within like sides, so is A. equall to D. B. to E. and C. to F. for thei are contained within like sides, as before is saied.



¶The .vj. Theoreme.

When any right line standeth on an other, the twoo angles that thei make, either are bothe right angles, or els equall to twoo right angles.

¶Example.

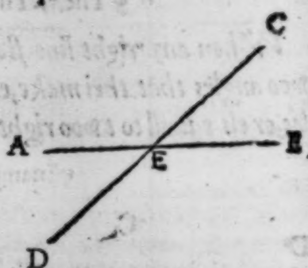
A. B. is a right line, and on it there doeth light an other right line, drawen frō C. perpendicularely on it, therefore saie I, that the twoo angles that thei doe make, are twoo right angles, as maie bee iudged by the definition of a right angle.

gle. But in the seconde parte of the example, where A. B, beyng still the right line, on whiche D. standeth in slope waies the twoo angles that be made of them, are not right angles, but yet thei are equall to twoo right angles, for so muche as the one is to greate, more then a right angle, so muche iuste is tother to little, so that bothe together are equall to twoo right angles, as you maie perceiue.

¶The. vij. Theoreme.

If twoo lines bee drawen to any one pricke in an other line, and those twoo lines dooe make with the first line, twoo right angles, either suche as bee equall to twoo right angles, and that towarde one hande, that those twoo lines doe make one straight line.

¶Example.



A. B. is a straighte line, on whiche there doth light twoo other lines one from D. and the other from C. but consideryng that thei meete in one pricke E. and that the angles on one hande bee equall to twoo right corners (as the last theoreme dooeth declare) therefore maie D. E. and E. C. bee coumpted for one right line.

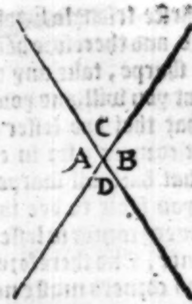
¶The. viij. Theoreme.

When twoo lines dooe cutte one an other crossewaies, thei dooe make their matche angles equall.

Example.

Example.

What matche angles are, I haue tolde you in ẏ definitions of the termes. And here A, and B, are matche corners in this example, as are also C. and D. so that the corner A. is equall to B. and the angle C. is equall to D.

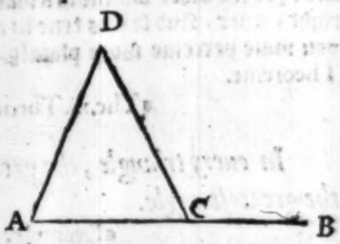


¶The.ix.Theoreme.

VVhen so euer in any triangle, the line of one side is drawen forthe in lengthe, that vtter angle is greater then any of the ij inner corners that ioyne not with it.

¶Example.

The triangle A.D.C. hath his ground line A.C. drawen foorthe in lengthe unto B, so that the vtter corner that it maketh in C. is greater then any of the twoo inner corners that lye against it, and ioygne not with it,



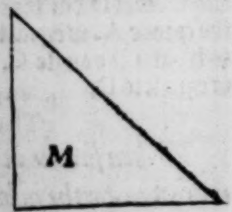
whiche are A. and D. for thei bothe are lesse then a right angle, and bee sharpe angles, but C. is a blunte angle, and therefore greater then a right angle.

¶The.x.Theoreme.

In euery triangle any twoo corners, how so ener you take them, are lesse then twoo right corners.

Example.

In the firste triangle E, whiche is a threlike, and therefore hath all his angles sharpe, take any twoo corners that you will, and you shal perceiue that thei bee lesser then twoo right corners, for in euery triangle that hath all sharpe corners (as you see it to bee in this example) euery corner is lesse then a right corner. And therefore, also euery twoo corners muste needes bee lesse then twoo right corners. Furthermore in that other triangle marked with M. whiche hath twoo sharpe corners, and one right, any two of them also are lesse then twoo right angles. For though you take the right corner for one, yet the other whiche is a sharpe corner, is lesse then a right corner. And so it is true in all kindes of triangles, as you maie perceiue more plainly by the twentie and twoo Theoreme.

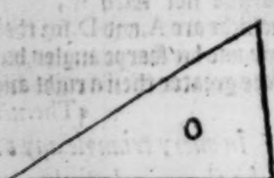


¶The.xj. Theoreme.

In euery triangle, the greatest side lieth againste the greatest angle.

¶Example.

As in this triangle A.B.C. the greateste angle is C. And A.B (whiche is the side that lieth againste it) is the greatest and lögest side. And contrariwaies is A.C. is the shortest line, so B. (whiche is the angle lying against it) in the smallest and sharpest



angle,

angle,for this doeth folowe also,that as the longest side lieth against the greatest angle,so it that foloweth.

¶The.xij. Theoreme.

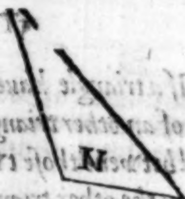
In euery triangle,the greatest angle lieth against the longest side.

For these twoo Theoremes are one in truthe.

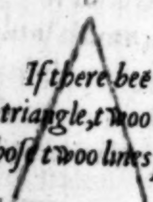
¶The.xiij. Theoreme.

In euery triangle any twoo sides together, howe so euer you take them,are longer then the thirde.

For example, you shall take this triangle A.B.C, whiche hath a verie blunte corner, and therefore one of his sides greater a good deale, then any of the other, and yet the twoo lesser sides together are greater then it. And if it bee so in a blunte angeled triangle, it muste needes be true in all other, for there is no other kinde of triangles,that hath the one side so greate aboue the other side, as thei that haue blunte corners.



¶The.xiiij.Theoreme.

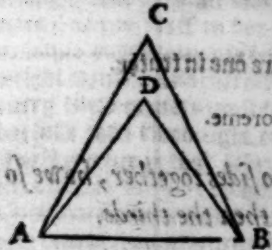


If there bee drawen from the endes of any side of a triangle,twoo lines meetyng within the the triangle, those twoo lines shall be lesse then the other twoo sides

 of

of the triangle, but yet the corner, that thei make, shall bee greater then that corner of the triangle, whiche standeth ouer it.

¶ Example.



A, B, C. is a triangle, on whose ground line A.B. there is drawen twoo lines from the twoo eandes of it, I saie from A. and B. and thei meete within the triangle, in the poincte D. wherefore I saie, that as those twoo lines A.D and B.D. are lesser then A.C. and B.C. so the angle D. is greater then the angle C. whiche is the angle against it.

¶ The. xv. Theoreme.

If a triangle haue twoo sides, equall to the twoo sides of an other triangle, but yet the angle that is contained betwene those twoo sides, greater then the like angle in the other triangle, then is his grounde line greater than the grounde line of the other triangle.

¶ Example.

A.B.C, is a triangle, whose sides A.C. and B.C. are equall to E.D. and D.F. the twoo sides of the triangle D.E.F. but because the angle in D. is greater then the angle C. whiche are the twoo angles contained betwene the equall lines, therefore

fore muste the grounde line E. F. nedes be greater thenne the grounde line A. B. as you see plainly.

¶The.xvj. Theoreme.

If a triangle haue twoo sides equall to the twoo sides of an other triangle, but yet hath a longer grounde line then that other triangle, then is his angle that lieth betwene the equall sides, greater then the like corner in the other triangle.

¶Example.

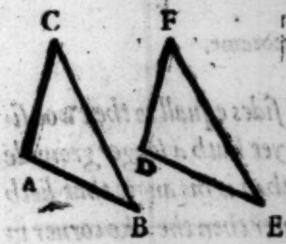
This Theoreme is nothyng els, but the sentence of the laste Theoreme tourned backewarde, and therefore needeth none other proofe, neither declaration, then the other example.

¶The.xvij. Theoreme.

If twoo triangles bee of suche sorte, that twoo angles of the one, bee equall to twoo angles of the other, and that one side of the one, bee equall to one side of the other, whether that side dooe adioyne to one of the equall corners, or els lye against one of them, then shall

the other twoo sides of those triangles bee equall toge- ther, and the thirde corner shall bee equall to those twoo triangles.

¶Example.



Because that A.B.C, thone triangle hath twoo corners A. and B, equall to D. E, that are twoo corners of the other tri- angle. D. E. F. and that thei haue one side in the bothe equall, that is A. B, whiche is equall to D. E. therefore shall bothe the other twoo sides bee equall one to an other, as A.C. and B.C. equall to D. F. and E. F. and also the third angle in them bothe shall bee equall, that is, the angle C, shall bee equall to the angle E.

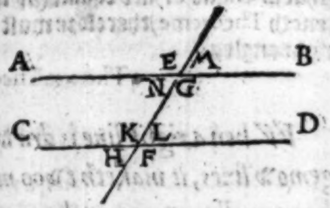
¶The.xviij. Theoreme.

VV hen on twoo righte lines there is drawen a thirde right line crossewaies, and maketh twoo matche corners of the one line equall to the like twoo matche corners of the other line, then are those twoo lines gem- mowe lines, or paralleles.

¶Example.

The twoo first lines are A. B. and C. D, the thirde line that crosseth them is E. F. And because that E. F. maketh twoo matche

matche angles with A. B. equall to twoo other like matche angles on C D. (that is to saie, E. G. equall to K.F. and M.N, equall also to H.L.) therfore are these twoo lines A.B. and C. D. gemowe lines, understande hereby like matche corners, those that goe one waie, as doeth E. G. and K.F. likewaies N. M. and H. L. for as E.G. and H. L. either N.M. and K.F. goe not one waie, so bee not thei like matche corners.



¶The xix. Theoreme.

When on twoo right lines there is drawen a third right line crossewaies, and maketh the twoo outer corners toward one hande equall together, then are those twoo lines paralleles. And in like maner of twoo inner corners toward one hand, be equall to two right angles.

¶Example.

As the Theoreme dooeth speake of twoo outer angles, so muste you understande also of twoo nether angles, for the iudgemente is like in bothe. Take for example the figure of the laste Theoreme, where A. B. and C. D. bee called paralleles also, because E. and K. (whiche are twoo outer corners) are equall, and like waies L. and M. And so are in like maner the nether corners N. and H. and G. and F. Nowe to the seconde part of the Theoreme, those twoo lines A.B. and C.D. shall be called paralleles, because the twoo inner corners. As for example, those twoo that bee towarde the right hande

(that is G. and L.) are equall (by the firste parte of this nineteneth Theoreme) therefore must G. and L. be equall to twoo right angles.

¶The.xx.Theoreme.

When a right line is drawen crosse ouer twoo right gemow lines, it maketh twoo matche corners of the one line, equall to twoo matche corners of the other line, and also bothe ouer corners of one hande equall together, and bothe nether corners likewaies, and more ouer twoo inner corners, and twoo vtter corners also towarde one hande, equall to twoo right angles.

¶Example.

Because A.B. and C.D. (in the last figure) are paralleles, therefore the twoo matche cornes of the one line, as E.G. be equall vnto twoo matche corners of the other line, that is K. F, and likewaies M.N. equall to H.L. And also E. and K. both ouer corners of the left hande equall together, and so are M. and L. the twoo ouer corners on the right hande, in like maner N. and H. the twoo nether corniers on the lefte hande, equall eche to other, and G. and F. the twoo nether angles on the right hande equall together.

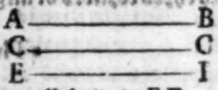
Farthermore, yet G. and L. the twoo inner angles on the right hande, bee equall to twoo right angles, and so are M. and F. the twoo vtter angles on the same hande, in like maner shall you saie of N. and K. the twoo inner corners on the lefte hande, and of E. and H. the twoo vtter corners on the same hande. And thus you see the agreable sentence of these three Theoremes to tende to this purpose, to declare by the angles how to iudge paralleles, and contrary waies how you maie by paralleles iudge the proportion of the angles.

The

¶The.xxj.Theoreme.

What so euer lines bee paralleles to any other line, those same bee paralleles together.

¶Example.

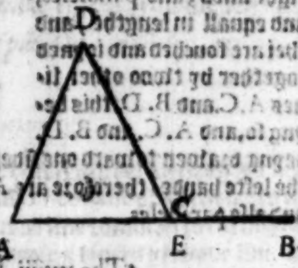


A.B. is a gemowe line, or a parallele vnto C.D. And E.F. likewaies is a parallele vnto C. D. Wherefore it foloweth, that A.B. must needes bee a parallele vnto E.F.

¶The.xxij.Theoreme.

In euery triangle, when any side is drawen foorthe in length, the vtter angle is equall to the twoo inner angles that lye against it. And all three inner angles of any triangle, are equall to twoo right angles.

¶Example.



The triangle beyng A.D.E. and the side A.E. drawen foorthe vnto B. there is made an vtter corner, whiche is C, and the vtter corner C, is equall to bothe the inner corners that lye against it, whiche are A. and D. And all their inner corners, that is to saie, A. D. and E. are equall to twoo right corners, whereof it foloweth, that all the three corners of any one triangle, are equall to all the three corners of euery other triangle. For what so euer thynges are equall to a-

ny

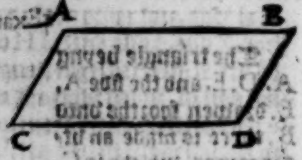
by one thirde thyng, those same are equall together, by the first common sentence, so that because all the thre angles of euery triangle, are equall to twoo right angles, and all right angles bee equall together (by the fowerth requeste) therefore muste it needes followe, that all the three corners of euery triangle (accoumptyng theim together) are equall to three corners of any other triangle, taken all together.

¶The.xxiij. Theoreme.

VVhen any twoo right lines doeth touche, and couple twoo other right lines, whiche are equall in lengthe, and paralleles, and if those twoo lines bee drawen towarde one hande, then are thei also equall together, and paralleles.

¶Example.

A.B, and C.D. are twoo right lines, and paralleles, and equall in lengthe, and thei are touched and ioyned together by twoo other lines A.C. and B.D. this beyng so, and A.C. and B.D. beyng drawen toward one side (that is to saie, bothe toward the lefte hande) therefore are A.C. and B.D. bothe equall, and also paralleles.



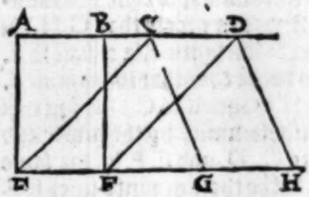
¶The.xxiiij. Theoreme.

In any likeiamme the twoo contrary sides are equall together, and so are eche twoo contrary angles,

and

and the bias line that is drawen in it, dooeth deuide it into twoo equall portions.

¶Example.



Here are twoo likeiammes ioigned together, the one is a long square A.B.F.E. and the other is a losengelike. D.C.E.F. whiche twoo likeiammes are proued equall together, because thei haue one grounde line, that is, F, E. And are made betwene one paire of gemowe lines, I meane A.D. and E.H. By this Theoreme maie you knowe the Arte of the righte measuryng of likeiammes, as in my booke of measuryng, I will more plainly declare.

¶The.xxvj.Theoreme.

All likeiammes that haue equall grounde lines, and are drawen betwene one paire of paralleles, are equall together.

¶Example.

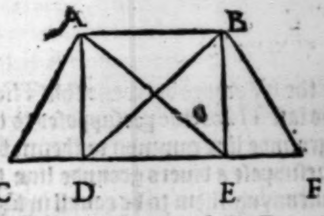
First you must marke the difference betwene this Theoreme and the laste, for the last Theoreme presupposed to the diuers likeiammes, one grounde line common to theim, but this Theoreme dooeth presuppose a diuers grounde line for euery likeiamme, onely meanyng them to be equall in lẽgth though thei bee diuers in noumber. As for example. In the laste figure there are twoo paralleles, A.D. and E.H. and betwene them are drawen three likeiammes, the firste is, A. B, E, F: the seconde is E.C, D, F: and the thirde is C, G, H, D

The first and the seconde haue one ground line, (that is E.F) and therefore, in so muche as thei are betwene one paire of paralleles, thei are equall accordyng to the fiue and twentie Theoreme, but the thirde likeiamme, that is C.G.H.D. hath his ground line G.H, seuerall from the ot her, but yet equall unto it. Wherefore the thirde likeiamme, is equall to the other twoo firste likeiammes. And for a proofe that G.H, beyng the grounde line of the thirde likeiamme, is equall to E.F. whiche is the ground line to bothe the other likeiammes, that maie be thus declared, G,H. is equall to C.D. seyng thei are the contrary sides of one likeiamme (by the fower and twentie Theoreme) and so are C.D. and E.F. by the same Theoreme. Therefore, seyng bothe those groundes lines E.F. and G.H. are equall to one third line (that is C.D.) thei must nedes be equall together by the first common sentence.

¶The.xxvij. Theoreme.

All triangles hauyng one grounde line, and standing betwene one paire of paralleles, are equal together.

¶Example.



A.B. and C.F. are twoo gemowe lines, betwene whiche there bee made twoo triangles, A.D.E. and D.E.B. so that D.E. is the common grounde line to the bothe, wherfore it doeth folowe, that those twoo triangles A.D. E. and D.E. B. are equall eche to other.

¶The.xxviij. Theoreme.

All

All triangles that haue like long grounde lines, and bee made betwene one paire of gemowe lines, are equall together.

¶Example.

Example of this Theoreme, you maie see in the laste figure, where as sixe trianges made betwene those twoo gemowe lines A.B.and C.F, the firste triangle is A. C. D, the seconde is A.D.E:the third is A.D.B:the fowerth is A.B.E. the fifte is D.E.B.the sixte is B.E.F.of whiche sixe triangles A,D.E.and D. E. B. are equall, because thei haue one common ground line.And so likewise A.B.E.and A.B.D.whose common grounde line is A.B.but A.C.D.is equall to B.E.F beyng bothe betwene one couple of paralleles, not because thei haue one grounde line, but because thei haue their grounde lines equall,for C.D. is equall to E. F. as you maie declare thus.C.D.is equall to A.B.(by the fower and twentie Theoreme) for thei are twoo contrary sides of one likesamme.A.C.D.B.and E,F. by the same Theoreme, is equall to A . B , for thei are the twoo the contrary sides of the likesamme,A.E.F.B, wherefore C.D.must néedes bee equall to E.F.likewise the triangle A.C.D,is equal to A.B.E.because thei are made betwene one paire of paralleles, and haue their grounde lines like,I meane C.D.and A.B. Againe A. D.E. is equall to eche of them bothe,for his grounde line D. E,is equall to A.B.in so muche as thei are the contrary sides of one likeiame,that is the long square A.B.D.E. And thus maie you proue the equalnesse of all the reste.

¶The.xxix.Theoreme.

All equall triangles that are made on one ground line, and rise one waie, muste needes bee betwene one paire of paralleles.

¶Example.

Take for example A. D. E, and D. E. B. whiche (as the twentie and seuen conclusion doeth proue) are equall together, and as you see, thei haue one grounde line D.E. And againe thei rise towarde one side, that is to saie, upwarde towarde the line A.B, wherefore thei muste needes bee inclosed betwene one paire of paralleles, whiche are here in this example A.B. and D.E.

¶The.xxx. Theoreme.

Equall triangles that haue their grounde lines equall, and bee drawen towarde one side, or made betwene one paire of paralleles.

¶Example.

The example that declareth the laste Theoreme, maie well serue to the declaration of this also. For those twoo Theoremes doe differ but in one poincte, that the last Theoreme meaneth of triangles, that haue one grounde line common to them both, and this Theoreme doeth presuppose the groundlines to be diuers, but yet of one length, as A.C.D. and B.E.F, as thei are twoo equall triangles approued, by the eight and twentie Theoreme, so in the same Theoreme it is declared, that their grounde lines are equall together, that is C.D. and E.F, now this beyng true, and consideryng that thei are made towarde one side, it followeth, that thei are made betwene one paire of paralleles, when I saie, drawen toward one side, I meane that the triangles must be drawē either bothe upward frō one parallele, either els bothe down ward, for if the one be drawen upward, and the other down warde, then are thei drawen betwene twoo paire of paralleles, presupposyng one to bee drawen by their grounde line, and then doe thei rise toward contrary sides.

¶The.xxxj.Theoreme.

If a likeiamme haue one grounde line with a triangle, and be drawen betwene one paire of paralleles, then shall the likeiamme be double to the triangle.

¶Example.

A.H. and B. G. are two gemowe lines, betwene whiche there is made a triāgle B.C.G. and a likeiamme A.B. G. C, whiche haue a grounde line that is to saie, B.G. Therefore dooeth it followe, that the likeiamme A.B.G. C, is double to the triangle B. C. G. For euery halfe of that likeiamme is equall to the triangle, I meane A.B. F. E. either F.E.C.G. as you maie conjecture by the. xi. conclusion Geometricall.

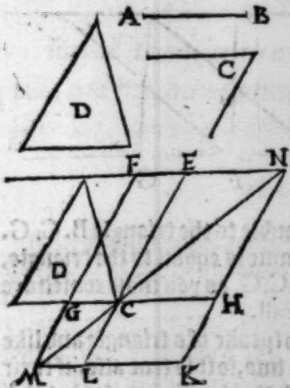
And as this Theoreme doeth speake of a triangle and likeiamme, that haue one grounde line, so it is true also, if their grounde lines bee equall, though thei bee diuers, so that thei bee made betwene one paire of paralleles. And hereof maie you perceiue the reason, why in measuryng the platte of a triangle, you muste multiplie the perpendicular line by half the grounde line, or els the whole grounde line by halfe the perpendicular, for by any of these bothe waies, is there made a likeiamme equall to halfe suche a one, as should bee made on the same whole grounde line with the triangle, and betwene one paire of paralleles. Therefore as that likeiamme is double to the triangle, so the halfe of it, muste needes bee equall to the triangle. Compare the eleuenth conclusion with this Theoreme.

 The

¶The.xxxij. Theoreme.

In all likeiammes, where there are more then one made aboute one bias line, the fill squares of euery of them must needes bee equall.

¶Example.



Bias lyne.

First, before I declare the examples, it shall bee meete to shewe the true vnderstandyng of this Theoreme. Therefore by the Bias line, I meane that line, whiche in any square figure doeth run from corner to corner. And euery square whiche is deuided by that bias line, into equall halfes from corner to corner (that is to saie, into twoo equall triangles) those bee compted to stande about one bias line, and the other squares, whiche touche that bias line, with one of their corners onely, those doe I call Fill squares, accordyng to the Greeke name, whiche is anapleromata, and called in Latine supplementa, because thei make one generall square, includyng and enclosyng the other diuers squares, as in this exāple H.C.E.N. is one square likeiamme, and L.M.G.C. is an other, whiche bothe are made aboute one bias line, that is, N.M. then K.L.H.C. and C.E.F.G. are twoo fill squares, for thei doe fill vp the sides of the twoo first square likeiammes, in suche sorte, that of all them fower is made one greate generall square K.M.F.N.

Fyll squares. ἀναπληρώματα.

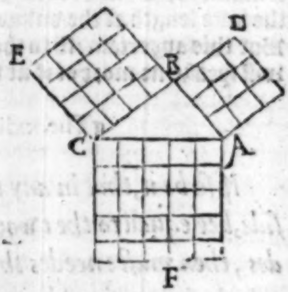
Now to the sentence of the Theoreme, I saie, that the twoo

twoo fill squares; H.K.L.C. and C.E.F.G. are bothe equall together, (as it shall bee declared in the booke of proofes) because thei are the fill squares of twoo likeiammes, made aboute one bias line, as the example sheweth. Conferre the twelfth conclusion with this Theoreme.

¶The.xxxiij.Theoreme.

In all right angeled triangles, the square of that side, whiche lieth against the right angle, is equall to the twoo squares of bothe the other sides.

¶Example.



A.B.C. is a triangle, hauyng a righte Angle in B. Wherfore it foloweth, that the square of A.C. (whiche is the side that lieth against the right angle) shall bee as muche as the twoo squares of A.B. and B.C. whiche are the other twoo sides.

By the square of any line, you muste understande a figure made iuste square, hauyng all his fower sides equall to that line, whereof it is the square, so is A.C.F, the square of A.C. Likewaies A.B.D. is the square of A.B. And B.C.E. is the square of B.C. Nowe by the nomber of the deuisions in eche of these squares, maie you perceiue not onely what the square of any line is called, but also that the Theoreme is true, and expressed plainly both by lines and number. For as you se, the greater square (that is A.C.F) hath v. deuisiõs on eche side, all equall together, and those in the whole square are

are.xxv. Now in the left square, whiche is A.B.D. there are but thre of those deuisions in one side, and that yeldeth nine in the whole. So likewaies you see in the meane square A.C.E. in euery side fower partes, whiche in the whole amoũt unto sixtene. Now adde together all the partes of the twoo lesser squares, that is to saie, sixtene and nine, and you perceiue that thei make twentie and fiue, whiche is an equall number to the somme of the greater square.

By this Theoreme you maie understande a readie waie, to knowe the side of any right angeled triangle that is unknowen, so that you knowe the lengthe of any twoo sides of it. For by tournyng the twoo sides certaine into their squares, and so addyng them together, either subtracting the one from the other (accordyng as the use of these Theoremes I haue set forthe) and then findyng the roote of the square that remaineth, whiche roote (I meane the side of the square) is the iuste length of the unknowen side, whiche is sought for. But this appertaineth to the thirde booke, and therefore I will speake no more of it at this tyme.

¶The.xxiiij. Theoreme.

If so be it, that in any triangle, the square of the one side, bee equall to the twoo squares of the other twoo sides, then muste needes that corner bee a right corner, whiche is contained betwene those twoo lesser sides.

¶Example.

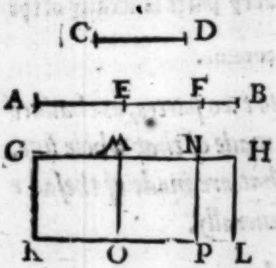
As in the figure of the last Theoreme, because A.C. made in square, is as muche as the square of A.B. and also as the square of B.C. ioyned bothe together, therfore the angle that is inclosed betwene those twoo lesser lines, A.B. and B.C. (that is to saie) the angle B. which lieth against the line A.C. must nedes be a right angle. This Theoreme doeth so depẽde of the truthe of the laste, that when you perceiue the truthe of

of the one, you can not iustly doubte of the others truthe, for thei containe one sentence, contrary waies pronounced.

¶The.xxxv. Theoreme.

If there bee sette forthe twoo right lines, and one of them parted into sundrie partes, how many or fewe so euer thei be, the square that is made of those two right lines proposed, is equall to all the squares, that are made of the vndeuided line, & euery part of the deuided line.

¶Example.



The twoo lines proposed, are A. B, and C. D, and the line A.B. is deuided into thre partes by E. and F. Now saith this Theoreme, that ẏ square that is made of those ij. whole lines A. B. and C. D, so that the line A. B. standeth for the length of the square, and the other line C.D. for the bredth of the same. That square (I saie) will be equall to al the squares, that be made, of the vndeuided line (whiche is C.D.) and euery portion of the deuided line. And to declare that particularly: Firste, I make an other line G. K. equall to the line C.D. and the line G.H. to be equall to the line A.B. and to be deuided into three like partes, so that G.M. is equall to A.E. and M.N equall to E.F, and then must N.H. nedes remaine equall to F.B. Then of those twoo lines G.K. vndeuided, and G.H. whiche is deuided, I make a square, that is G.H.K.L. In whiche square if I drawe crosse lines from one side to the other, accordyng to the deuisions of the line G.A. then will it

 appere

appere plain, that the Theoreme doeth affirme. For the first square G.M.O.K, must nedes be equall to the square of the line C. D, and the first portion of the deuided line, whiche is A. E, for because their sides are equall. And so the seconde square that is M.N.P.O. shal be equall to the square of C.D. and the second part of A.B, that is E.F. Also the third square whiche is N.H.L.P, must of necessitie be equal to the squars of C.D, and F.B, because those lines be so coupled that euery couple are equall in the seuerall figures. And so shall you not onely in this example, but in al other finde it true, that if one line bee deuided into sondrie partes, and an other line whole and vndeuided, matched with hym in a square, that square whiche is made of these twoo whole lines, is as muche iuste and equally, as all the seuerall squares, whiche bee made of the whole line vndeuided, and euery parte seuerally of the deuided line.

¶The. xxxvj. Theoreme.

If a right line be parted into twoo partes, as chaunce maie happe, the square that is made of that whole line, is equall to bothe the squares that are made of the same line, and the twoo partes of it seuerally.

¶Example.

The line propouned beyng A. B. and deuided, as chaunce happeneth, in C, into twoo vnequall partes, I saie that the square made of the whole line A. B. is equall to the twoo squares made of the same line, with the twoo partes of it self, as with A. C. and with C. B, for the square D. E. F. G. is equall to the twoo other partiall squares of

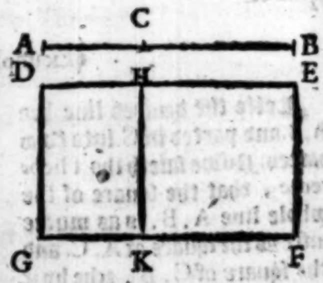
D.H

D.H.K.G. and H.E.F.K, but that the greater square is equal to the square of the whole line A.B. & the partial squares equall to the squares of the seconde partes of the same line, ioyned with the whole line, your eye maie iudge without muche declaration, so that I shall not nede to make more exposition therof, but that you maie examine it, as you did in the laste Theoreme.

¶The.xxxvij.Theoreme.

If a right line bee deuided by chaunce, as it maie happen, the square that is made of the whole line, and one of the partes of it, whiche so euer it bee, shall bee equall to that square that is made of the twoo partes ioyned together, and to an other square made of that parte, whiche was before ioyned with the whole line.

¶Example.



The line A.B. is deuided in C. into twoo partes, though not equally, of whiche twoo partes, for an example I take the firste, that is A.C. and of it I make one side of a square, as for example D.G. accomptyng those twoo lines to bée equal, the other side of the square is D.E. whiche is equall to the whole line A.B.

Now maie it appeare to your eye, that the greate square made of the whole line A.B, & of one of his parts that is A.C

(whiche is equall with D.G.) is equall to twoo partiall squares, whereof the one is made of the saied greater portion A.C. in as muche as not onely D.G. beyng one of his sides, but also D.H. beyng the other side, are eche of theim equall to A.C. The seconde square is H.E.F.K, in whiche the one side H.E. is equall to C.B. beyng the lesser parte of the line A.B. and E.F. is equall to A.C. whiche is the greater parte of the same line. So that those twoo squares D.H.K.G. and H.E.F.K. be bothe of them no more then the greater square D.E.F.G. accordyng to the wordes of the Theoreme afore saied.

The. xxxviij. Theoreme.

If a righte line bee deuided by chaunce, into partes, the square that is made of that whole line, is equall to bothe the squares that are made of eche parte of the line, and more ouer to twoo squares made of the one portion of the deuide line ioygned with the other in square.

Example.

Lette the deuided line bee A.B. and parted in C. into twoo partes: Nowe saieth the Theoreme, that the square of the whole line A.B. is as muche iuste as the square of A.C. and the square of C.B. eche by it selfe, and more ouer by as muche twise, as A.C. and C.B. ioyned in one square will make

For as you see, the greate square D.E.F.G, conteineth in hym fower lesser squares, of whiche the firste and the greatest is N.M.F.K. and is equall to the square of the line A.C. The seconde square is the least of them all, that is D.H.L.N, and it is equall to the square of the line C.B. Then are there twoo other long squares bothe of one bignesse, that is H.E.N.M. and L.N.G.K. eche of them bothe hauyng twoo sides equall to A.C. the longer parte of the deuided line, and there other twoo sides equall to C.B. beyng the shorter part of the saied line A.B.

So is that greatest square, beyng made of the whole line A.B, equall to the twoo squares of eche of his partes seuerally, and more by as muche iuste as twoo longe squares, made of the longer portion of the deuided line, ioyned in square with the shorter parte of the same deuided line, as the Theoreme would. And as here I haue putte an example of a line deuided into twoo partes, so the Theoreme is true of all deuided lines, of what number so euer the partes bee, fower fiue, or sixe. &c.

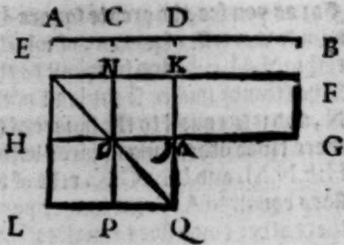
This Theoreme hath greate use, not onely in Geometrie but also in Arithmetike.

¶ The.xxxix. Theoreme.

If a right line bee deuided into twoo equall partes, and one of these twoo partes deuided againe into twoo other partes, as happeneth the long square that is made of the thirde, or later parte of that deuided line, with the residue of the same line, and the square of the midle moste parte, are bothe together equall to the square of halfe the first line.

¶ Example.

 The



The line A.B. is deuided into twoo equall partes in C, and that parte C. B. is deuided againe as happeneth in D. Wherfore saieth the Theoreme, that the longe square made of D. B. and A. D, with the square of C.D. (whiche is the middle portion) shall bothe bée equall to the square of halfe the line A. B. that is to saie, to the square of A. C. or els of C. D. whiche make all one. The longe square F.G.N.O. whiche is the longe square that the Theoreme speaketh of, is made of twoo long squares, wherof the firste is F.G.M.K, and the seconde is K.N.O.M. The square of the middle portion is L.M.O.P. And the square of the halfe of the firste line is E.K.Q.L. Nowe by the Theoreme, that long square F. G. M. O. with the iuste square L. M. O. P, muste bee equall to the greate square E. K. Q. L. whiche thyng because it seemeth somewhat difficult to vnderstande, although I intende not here to make demonstrations of the Theoremes, because it is appoincted to bee doen in the newe edition of Euclide, yet I will shewe you briefly how the equalitie of the partes doeth stande. And first I saie, that where the comparison of equalitie is made, betwene the greate square (whiche is made of halfe the line A.B.) and twoo other, whereof the firste is the long square F.G.N.O. and the second is the full square L. M. O. P, whiche is one portion of the greate square all readie, and so is that longe square K.N. M. O. beyng a parcell also of the longe square. F.G.N.O. Wherefore as those twoo partes are common to bothe partes compared in equalitie, and therefore beeyng bothe abated from eche parte, if the reste of bothe eche other partes bee equall, then were those whole partes equall before: Nowe the reste of the greate square, those twoo lesser

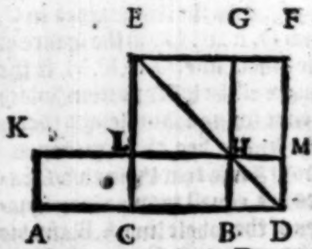
ser squares beyng taken awaie, is that long square E.N.P.Q. whiche is equall to the long square F.G.K.M. being the rest of the other parte. And that thei twoo bee equall, their sides doe declare. For the longest lines that is F.K. and E.Q. are equall, and so are the shorter lines, F, G, and E, N. and so appeareth the truthe of the Theoreme.

¶The.xl.Theoreme.

If a right line bee deuided into twoo euen partes, and an other right line annexed to one ende of that line, so that it make one right line with the firste. The long square that is made of this whole line so augmented, and the portion that is added, with the square of halfe the right line, shall bee equall to the square of that line, whiche is compounded of halfe the firste line, and the parte newly added.

¶Example.

The firste line propouned is A.B, and it is deuided into twoo equall partes in C, and an other right line, I meane B.D. annexed too one ende of the firste line.



Nowe saie I, that the long square A.D.M.K. is made of the whole line so augmented, that is A.D, and the portion annexed, the is D.M, for D.M. is equall to B.D wherefore that long square A.D.M.K, with the square of halfe that first line, that is E.G.H.L, is equall to the greate square E.F.D.C, which square is made of ẏ line C.D, that is to

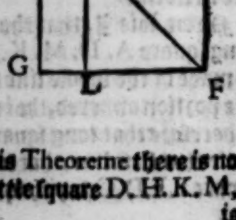
to saie, of a line compounded of halfe the firste line, beyng C. B. and the portion annexed, that is B. D. And it is easely perceiued, if you consider that the longest square A. C. L. K. (whiche onely is lefte out of the greate square) hath an other long square equall to hym, and so supplie his stede in the greate square, and that is G. F. M. H. For their sides bee of like lines in length.

¶The .xlj. Theoreme.

If a right line bee deuided by chaunce, the square of the same whole line, and the square of one of his partes, are iuste equall to the longe square of the whole line, and the saied parte twise taken, and more ouer to the square of the other parte of the saied line.

¶Example.

A. B. is the line deuided in C. And D. E. F. G. is the square of the whole line, D. H. K. M. is the square of the lesser portion (which I take for an example) and therefore muste bee twise reckened. Now I saie that those two squares are equall to twoo long squares of the whole line A. B. and his saied portion A. C. and also to the square of the other portion of the saied firste line, whiche portion is C. B. and his square K. N. F. L. In this Theoreme there is no difficultie, if you consider that the little square D. H. K. M, is



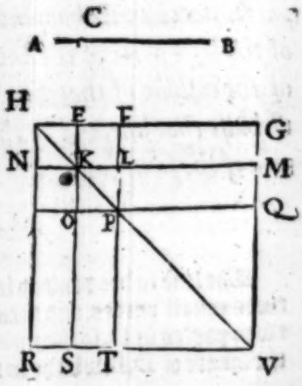
is fower tymes reckened, that is to saie, first of all, as a part of the greatest square, whiche is D. E. F. G. Secondly, he is reckened by hym self. Thirdly, he is accompted as parcell of the longe square D. E. N. M. And fourthly, he is taken as a parte of the other long square D.H.L.G, so that in as muche as he is twise reckened in one parte of the comparison of equalitie, and twise also in the seconde parte, there can rise none occasion of errour, or doubtfulnesse thereby.

¶The.xlij.Theoreme.

If a right line bee deuided as chaunce happeneth the fower long squares, that maie bee made of that whole line and one of his partes, with the square of the other parte, shall bee equall to the square that is made of the whole line, and the saied firste portion ioyned to hym in length, as one whole line.

¶Example.

The firste line is A.B. and is deuided by C. into twoo vnequall partes as happeneth, the long square of it, and his lesser portion A, C, is fower tymes drawen, the firste is E. G. M. K. the seconde is K. M. Q. O. the thirde is H. K. R. S. and the fowerth is K. L. S. T. And where as it appeareth that one of the little squares (I meane K. L. P. O.) is reckened twise,



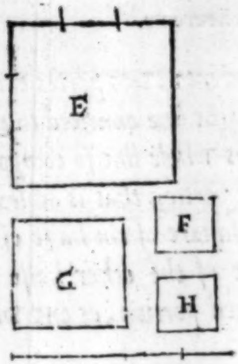
once as parcell of the seconde long square, and againe as part of the thirde longe square, to auoide ambiguitie, you maie place one in steede of it, an other square of equalitie with it, that is to saie, D.E.K.H, whiche was at no tyme accompanyng as parcell of any of theim, and then haue you fower longe squares distinctly made of the whole line A.B, and his lesser portion A. C. And within theim is there a greate full square P. Q. T. V. whiche is the iuste square of B. C. beeyng the greater portion of the line A. B. And that those fiue squares, dooe make iuste as muche as the whole square of that longer line D.G. (whiche is as long as A. B, and A.C. ioyned together) it maie bee iudged easily by the eye, sithe that one greate square dooeth comprehende in it all the other fiue squares, that is to saie, fower long squares (as is before mentioned) and one full square, whiche is the intents of the Theoreme.

¶The. xliij. Theoreme.

If a right line bee parted into twoo equall partes firste, and one of those partes againe into other twoo partes, as chaunce happeneth, the square that is made of the laste parte of the line so deuided, and the square of the residue of that whole line, are double the square of halfe that line, and to the square of the middle portion of the same line.

¶Example.

The line to bee deuided is A. B, and is parted in C. into twoo equall partes, and then C. B, is deuided againe into twoo partes in D, so the meanyng of the Theoreme, is that the square of D.B, whiche is the latter parte of the line, and the

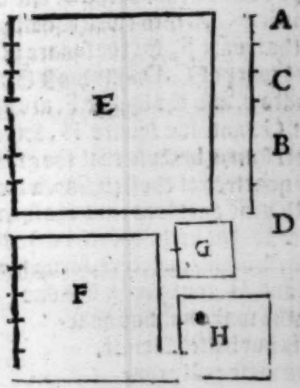


the square of A . D, whiche is the residue of the whole line. These twoo squares, I saie, are double to the square of the one halfe of the line, and too the square of C.D, whiche is ẏ middle portiō of those three deuisions. Whiche thing that you maie more easilie perceiue, I haue drawen fower Squares, whereof the greatest beeyng marked with E. is the square of A.D. The nexte, whiche is marked with G, is the square of halfe the line, that is, of A.C. And the other twoo little squares marked with F. and H, bee bothe of one bignesse, by reason that I did deuide C.B. into twoo equall partes, so that you maie take the square F. for the square of D. B, and the square H. for the square of C. D. Nowe I thinke you doubte not, but the square E. and the square F, are double so muche as the square G. and the square H, whiche thing the easier is to be vnderstande, because that the greate square hath in his side three quarters of the first line, whiche multiplied by it self, maketh nine quarters, and the square F. containeth but one quarter, so that bothe dooeth make ten quarters. Then G. containeth fower quarters, seyng his side containeth twoo, and H. containeth but one quarter, whiche bothe make but fiue quarters, and that is but halfe of tenne. Whereby you maie easily coniecture, that the meaning of the Theoreme is verified in the figures of this example.

¶The.xliiij. Theoreme.

If a right line bee deuided into twoo partes equally, and an other portion of a right line annexed to that firste line, the square of this whole line so compounded, and the square of the portion that is annexed, are double as muche as the square of the halfe of the firste line, and the square of the other halfe ioygned in one with the annexed portion, as one whole line.

¶Example.



The line is A.B, and is deuided first into two equall partes in C. and then is there annexed to it an other portion, whiche is B. D. Nowe saith the Theoreme, that the square of A. D. and the square of B. D. are double to the square of A. C. and to the square of C.D. The line A.B, containyng fower partes, then muste needes his halfe containe twoo partes, of suche partes I suppose B.D. (whiche is the annexed line) to containe three, so shall the whole line comprehende seuen partes, and his square fowertie and nine partes, whereunto if you adde the square of the annexed line, whiche maketh nine, then those bothe

bothe dooe yelde fiftie and eight, whiche must bee double to the square of the halfe line with the annexed portion. The halfe line by it self containeth but twoo partes, and therefore his square doeth make fower. The halfe line with the annexed portion containeth fiue, and the square of it is fiue and twentie, nowe putte fower to fiue and twentie, and it maketh iuste twentie and nine, the euen halfe of fiftie and eight, whereby it appeareth the truthe of the Theoreme.

¶The.xlv.Theoreme.

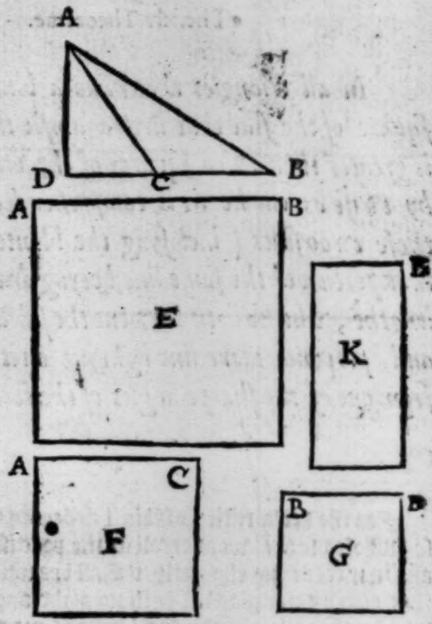
In all triangles that haue a blunte angle, the square of the side that lieth againste the blunte angle, is greater then twoo squares of the other twoo sides, by twise as muche as is comprehended of the one of those twoo sides (inclosyng the blunte corner) and that portion of the same line, beeyng drawen foorthe in lengthe, whiche lieth betwene the saied blunte corner, and a perpendiculare line lightyng on it, and drawen from one of the sharpe angles of the foresaied triangle.

¶Example.

For the decla ration of this Theoreme, and the nexte also, whose vse are wonderfull in the practise of Geometrie, and in measuryng especially, it shall bee needefull to declare that euery triangle that hath no right angle, as those bee whiche are called (as in the booke of practise is declared) sharpe cornered triangles, and blunte cornered triangles, yet maie thei bee brought to haue a right angle, either by partyng them into twoo lesser triangles, or els by addyng

an other triangle unto them, whiche maie be a greate helpe for the aide of measuryng, as more largelie shall bee sette foorthe in the booke of measuryng. But for this presente place, this fourme will I use, whiche Theon also useth) too adde one triangle unto an other, to bryng the blunte cornered triangle, into a right angled triangle, whereby the proportion of the squares of the sides in suche a blunte cornered triangle, maie the better bee knowen.

First therfore I sette forth the triangle A.B.C. whose corner by C. is a blūt corner, as you maye well iudge. then to make an other triangle, of it with a right angle, I muste drawe forth the side B. C. unto D. and from the sharpe corner by A. I bring a plumbe line or perpendiculare on D. And so is ther now a newe triangle A. B D. whose angle by D. is a right angle.



Now accordyng to the meanyng of the Theoreme, I saie, that in the firste triangle A. B. C. because it hath a blunte corner at C, the square of the

the

the line A. B. whiche lieth againste the saied blunte corner, is more then the square of the line A. C, and also of the line B.C, (whiche inclose the blunt corner) by as muche as will amounte twise of the line B.C, and that portion D. C. whiche lieth betwene the blunte angle by C, and the perpendicular line A.D.

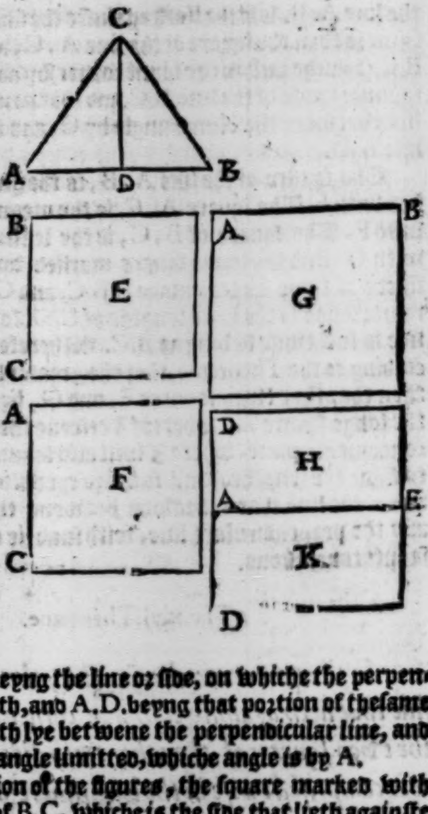
The square of the line A. B, is the greate square marked with E. The square A. C. is the meane square marked with F. The square of B. C, is the leaste square marked with G. And the longe square marked with K, is sette in steede of twoo squares made of B.C, and C. D. For as the shortest side is the iuste lengthe of C.D, so the other longer side is iust twise so long as B.C. Wherefore I saie now, accordyng to the Theoreme, that the greater square E, is more then the other twoo squares F. and G, by the quantitie of the longe square K. whereof I reserue the proofe to a more conueniente place, where I will also teache the reason how to finde the lengthe of all suche perpendiculare lines, and also of the line that is drawen betwene the blunte angle, and the perpendiculare line, with sundrie other verie pleasaunte conclusions.

¶The.xlvj.Theoreme.

In sharpe cornered triangles, the square of any side that lieth againste a sharpe corner, is lesser then the twoo squares of the other twoo sides, by as muche as is comprised twise in the long square of that side, on whiche the perpendiculare line falleth, and the portion of that same line, lying betwene the perpendiculare, and the foresaied sharpe corner.

¶Example.

First



First I set forthe the triangle A.B.C. and in it I drawe a plūbe line from the angle C. vnto the line A.B. and it lighteth in D. Nowe by the Theoreme, the square of B.C. is not so muche as the square of the other twoo sides, that of B.A. and of A.C. by asmuch as it is twise contained in the lōg square made of A.B. and A.D. A.B. beyng the line or side, on whiche the perpendicular line falleth, and A.D. beyng that portion of the same line, whiche doeth lye betwene the perpendicular line, and the saied sharpe angle limitted, whiche angle is by A.

For declaration of the figures, the square marked with E. is the square of B.C, whiche is the side that lieth against the sharpe angle, the square marked with C. is the square of A.B. and the square marked with F. is the square of A.C, and the twoo longe squares marked with H.K, are made of the whole line A.B, and one of his portions A.D. And truthe it is that the square E. is lesser then the other twoo squares C. and F. by the quantie of those twoo long squares H. and K.

Whereby

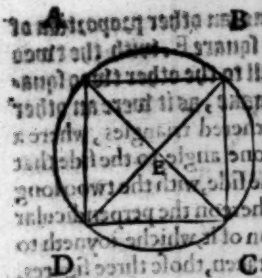
Whereby you maie consider againe, an other proportion of equalitie, that is to saie, that the square E. with the twoo longe squares H.K, are iuste equall to the other twoo squares C. and. F. And so maie you make, as it were an other Theoreme. That in all sharpe cornered triangles, where a perpendicular line is drawen from one angle, to the side that lieth against it, the square of any one side, with the twoo long squares made of that whole line, whereon the perpendicular line dooeth light, and of that portion of it, whiche ioyneth to that side, whose square is all readie taken, those three figures, I saie, are equall to the twoo squares, of the other twoo sides of the triangle. In whiche you muste understande, that the side on whiche the perpendiculare falleth, is thrise used, yet is his square but once mencioned, for twise he is taken for one side of the twoo long squares. And as I haue thus made as it were an other Theoreme out of this fowertie and sixe Theoreme, so mighte I out of it, and the other that goeth nexte before, make as many as would suffice for a whole booke, so that when thei shall bee applied to practise, and consequently to expresse their benefits, no manne that hath not well waighed their wonderfull commoditie, would credite the possibilitie of their wounderfull use, and large aide in knoweledge. But all this will I remitte to a place conuenient.

The.xlvij.Theoreme.

If twoo poinctes bee marked in the circumference of a circle, and a righte line drawen from the one to the other, that line muste needes fall with in the circle.

Example.

The circle is A.B.C.D, the twoo poinctes are A. B. the



right line that is drawen from the one to the other; is the line A. B, whiche as you see, muste needes lighte within the circle. So if you putte the poinctes to bee A. D, or D. C, or A. C, either B. C. or B. D. in any of these cases you see, that the line that is drawen from the one pricke, to the other, dooeth euermore runne within the edge of the circle, els can it bee no right line. How bee it, that a crooked line, especially beeyng more crooked then the portion of the circumference, maie bee drawen from poincte to poincte, without the circle. But the Theoreme speaketh onely of right rines, and not of crooked lines.

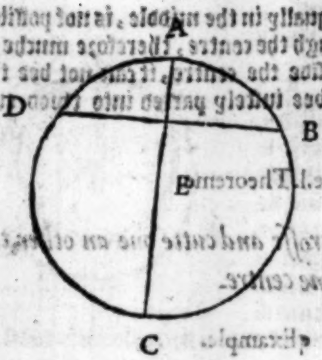
¶ The. xlviij. Theoreme.

If a righte line passynge the centre of a circle, dooe crosse an other righte line within the same circle, passyng beside the centre, if he deuide the saied line into twoo equall partes, then dooe thei make all their angles righte. And contrarie waies, if thei make all their angles right, then dooeth the longer line, cut the shorter in twoo partes.

¶ Example.

The circle is A.B.C.D, the line that passeth by the centre is A E. C, the line that goeth beside the centre is D. B. Nowe saie I, that the line A. E. C, dooeth cutte that other

line

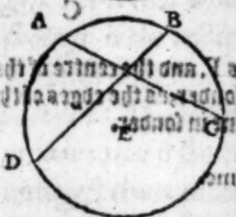


line D.B. into twoo iust partes, and therfore all their fower angles are right angles. And contrarie waies, because all their angles ar right angles, therefore it muste bee true, that the greater cutteth the lesser into twoo equall partes, accordyng as the Theoreme would.

¶The.xlix. Theoreme.

If twoo right lines drawen in a circle, doe crosse one an other, and dooe not passe by the centre, euery of them doeth not deuide the other into ij. equall portions.

¶Example.



The circle is A.B.C.D, and the centre is E, the one line A.C. and the other is B.D. whiche twoo lines crosse one an other, but yet thei goe not by the centre, wherefore accordyng to the woordes of the Theoreme, eche of them doeth cutte the other into equall portions. For as you maie easily iudge, A.C, hath one portion longer and an other shorter, and so like wise B.D. Howbeit, it is not so to be vnderstande, but one of thē maie be deuided into ij. euen parts,

but bothe to bee cutte equally in the middle, is not possible, unlesse bothe passe through the centre, therefore muche rather when bothe goe beside the centre, it can not bee that eache of theim should bee iustely parted into twoo euen partes.

¶The.l. Theoreme.

If twoo circles crosse and cutte one an other, then haue not thei bothe one centre.

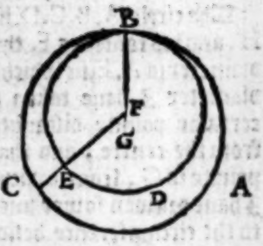
¶Example.

This Theoreme seemeth of it self so manifest, that it nedeth neither demonstration, neither declaration. Yet for the plaine understandyng of it, I haue set forthe a figure here, where twoo circles bee drawen, so that one of theim dooeth crosse the other (as you see) in the poinctes B. and G., and their centres appeare at the firste sighte to bee diuers. For the centre of the one is F. and the centre of the other is E. whiche differ as farre a sonder, as the edges of the circles, where thei bee moste distaunt in sonder.

¶The.lj. Theoreme.

If twoo circles bee so drawen, that one of them doe touche the other, then haue thei not one centre.

¶Example.

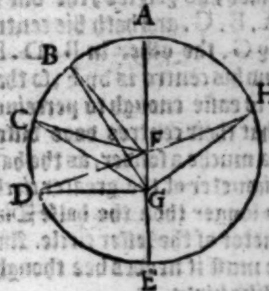


There are twoo Circles made, as you see, the one is A.B.C. and hath his centre by G. the other in B.D.E. and his centre is by F, so that it is easie enough to perceiue, that their centres dooe differ as muche a sonder, as the half diameter of the greater circle is longer then the halfe Diameter of the lesser circle. And so must it needes bee thought and saied of all other circles in like kinde.

¶The.lij. Theoreme.

If a certaine poincte bee assigned in the diameter of a circle, distaunte from the centre of the saied circle, and from that poincte diuerse lines drawen to the edge and circumference of the same circle, the longest line is that whiche passeth by the centre, and the shortest is the residewe of the same line. And of all the other lines that is euer the greatest, that is nighest to the line, whiche passeth by the centre. And contrary waies, that is shorteste, that is furthest from it. And emongest theim all there can bee but onely twoo equall together, and thei muste needes bee so placed, that the shorteste line shall bee in the iuste middle betwixte theim.

¶Example.



The circle A.B.C.D.E.H. and his centre is F, the diameter is A.E, in whiche diameter I haue taken a certaine poincte distaunte from the centre, and that poincte is G, from whiche I haue drawen fower lines to the circumference, beside the twoo partes of the diameter, whiche maketh vp sixe lines in al. Now for the diuersitie in quantitie of these lines, I saie, accordyng to the Theoreme, that the line whiche goeth by the centre is the longest line, that is to saie, A.G, and the residewe of the same diametre beeyng G.E, is the shortest line. And of all the other, that line is longeste, that is nerest vnto that parte of the diametre, whiche goeth by the centre, and that is shortest, that is farthest distaunte from it, wherefore I saie, that G.B, is longer then G.C, and therefore muche more longer then G.D, sithe G.C, also is longer then G.D, and by this maie you sone perceiue, that it is not possible to drawe twoo lines, on any one side of the diameter, whiche might bee equall in lengthe together, but on the one side of the diameter, maie you easily make one line equall to an other, on the other side of the same diametre, as you see in this example G.H, to bee equall to G.D, betwene whiche the line G.E, (as the shortest in all the circle) dooeth stande euen distaunte from eche of theim, and that is the precise knowledge of their equalitie, if thei bee equally distaunte from one halfe of the diameter. Where as contrary waies, if the one bee nerer to any one halfe of the diameter then the other is, it is not possible that thei twoo maie bee equall in length, namely if thei dooe ende bothe in the

the circumference of the circle, and bee bothe drawen from one poincte in the diametre, so that the saied poincte bee (as the Theoreme dooeth suppose) somewhat distaunt from the centre of the saied circle. For if thei bee drawen from the centre, then muste thei of necessitie bee all equall, howe many so euer thei bee, as the definition of a circle dooeth importe, without any regarde howe nere so euer thei bee to the diametre, or hawe distaunce from it. And here is to bee noted, that in this Theoreme, by neerenesse and distaunce is understande, the neerenesse and distaunce of the extreame partes of those lines, where thei touche the circumference. For at the other ende, thei dooe all meete and touche.

¶The.liij.Theoreme.

If a poincte bee marked without a circle, and from it diuerse lines drawen crosse the circle, to the circumference on the other side, so that one of theim passe by the centre, then that line whiche passeth by the centre, shall bee the longeste of all theim that crosse the circle. And of the other lines those are longeste, that bee nexte vnto it that passeth by the centre. And those are shorteste, that bee fartheste distaunte from it. But emonge those partes of those lines, whiche ende in the outwarde circumference, that is mooste shorteste, whiche is parte of the line that passeth by the centre, and emongeste the other eche of theim the nerer thei are vnto it, the shorte

ter thei are, and the farther from it, the longer thei bee. And emongeste theim all there can not bee more then twoo of any one in lengthe, and thei twoo muste be on the twoo contrary sides of the shortest line.

¶Example.

Take the circle to bee A B.C. and the poinct assigned without it to bee D. Now saie I, that if there bee drawen sunderie lines from D. and crosse the circle, endyng in the circumference on the contrarie side, as here you see, D.A, D.E, D.F, and D.B, then of all these lines, the longest must needes bee D.A, whiche goeth by the centre of the circle, and the nexte vnto it, that is D.E, is the longest emongest the reste. And contrary waies, D.B, is the shortest, because it is the farthest distaunt from D.A. And so maie you iudge of D.F, because it is nerer vnto D.A, then is D.B, therefore is it longer then D.B. And like waies because it is farther from D.A, then D.E, therefore is it shorter then D.E. Nowe for those partes of the lines, whiche bee without the circle (as you see) D.C, is the shorteste, because it is the parte of that line, whiche passeth by the centre. And D.K. is nexte to it in distaunce, and therfore also in shortnesse, so D.G. is farthest from it in distance, and therefore is the longest of theim. Now D.H. beyng nerer then D.G, is also shorter then it, and beeyng farther of, then

then D, K, is longer then it. So that for this parte of the Theoreme (as I thinke) you doe plainlie perceiue the truth thereof, so the residewe hath no difficultie. For seyng that the nerer any line is to D.C, (whiche ioyneth with the diameter) the shorter it is, and the farther of from it, the longer it is. And seyng twoo lines can not bee of like distaunce, beyng bothe on one side, therefore if thei shall be of one length, and consequentlie of one distaunce, thei muste needes bee on contrarie sides of the saied line D. C. And so appeareth the meanyng of the whole Theoreme.

And of this Theoreme dooeth there followe an other like, whiche you maie call, either a Theoreme by it self, or els a Corollarie vnto this laste Theoreme, I passe not so muche for the name. But his sentence is this: when so euer any lines bee drawen from any poincte, without a circle, whether thei crosse the circle, or eande in the vtter edge of his circumference, those twoo lines that bee equally distaunt from the leaste line are equall together, and contrary waies, if thei bee equall together, thei are also equally distaunt from that least line.

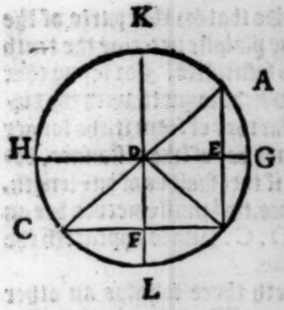
For the declaration of this proposition, it shall not neede to vse any other example, then that whiche is brought for the explication of this laste Theoreme, by whiche you maie without any teachyng easily perceiue, bothe the meanyng, and also the truthe of this proposition.

¶ The.liiij.Theoreme.

If a poincte bee sette in a circle, and from that poincte vnto the circumference many lines drawen, of whiche more then twoo are equall together, then is that poincte the centre of that circle.

¶Example.

 The



The circle is A.B.C, and within it I haue sette forthe for an example thrée prickes, whiche are D. E. and F, and frō euery one of them I haue drawen (at the leaste) fower lines vnto the circumference of the circle, but from D, I haue drawen more, yet maie it appeare readilie vnto your eye, that of all the lines whiche bee drawen from E. and F, vnto the circumference, there are but twoo equall, and more can not bee, for G. E. nor E. H, hath none other equall to theim, nor can not haue any, beeyng drawen from the same poincte E. No more can L. F. or F. K, haue any line equall to either of them, beyng drawen from the same point F. And yet from either of those twoo poinctes, are there drawen twoo lines equall together, as A. E, is equall to E. B, and B. F, is equall to F. C, but there can no thirde line bée drawen equall to either of these twoo couples, and that is, by reason that thei be drawen from a poincte distaunt from the centre of the circle. But from D, although there bee seuen lines drawen to the circumference, yet all bee equall, because it is the centre of the circle. And therefore if you drawe neuer so many more from it vnto the circumference, all shal bee equall, so that this is the priuilege (as it were of the centre) and therefore no other poincte can haue aboue twoo equall lines drawen from it vnto the circumference. And frō all poinctes you maie drawe twoo equall lines to the circumference of the circle, whether that poincte bee within the circle, or without it.

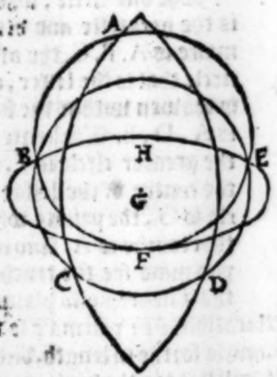
¶ The.lv. Theoreme.

No circle can cutte an other circle, in more poinctes

then twoo.

...rawen frō
...centre of
...cle, vnto
...tre of the

¶Example.

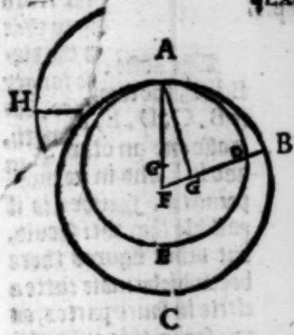


The firste ci... , dooeth B.F.E, the secō... nste ne- is B.C.D.E, ... te B.) crosse one an othe... ve and in E, and in no m... poinctes. Neither is it possible that thei should, but other figures there bee, whiche maie cutte a circle in foure partes, as you see in this example. Where I haue set forth one tunne fourme, and one eye fourme, and eche of them cutteth euery of their twoo circles into fower partes. But as thei be irregulare fourmes, that is to saie, suche fourmes as haue no precise measure, neither proportion in their draughte, so can there scarsely be made any certaine Theoreme of them. But circles are regulare fourmes, that is to saie, suche formes as haue in their protracture, a iuste and certaine proportion, so that certaine and determinate truthes maie bee affirmed of them, sithe thei are vniforme and vnchaungeable.

¶The.lvj.Theoreme.

If twoo circles bee so drawen, that the one be within the other, and that thei touche one an other: If a line bee drawen by bothe their centres, and so foorthe in length, that line shall runne to that poincte, where the circles dooe touche.

¶Example.



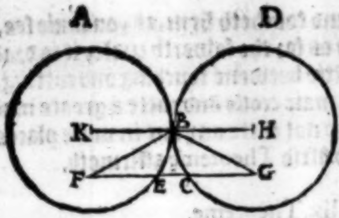
The one circle, whiche is the greateste and vttermoste is A. B. C, the other circle that is the lesser, and is drawen within the first, is A. D. E. The centre of the greater circle is F, and the centre of the lesser circle is G, the poincte where thei touche is A. And now you maie see the truthe of the Theoreme so plainely, that it needeth no farther declaration. For you maie se, that drawyng a line from F. to G, and so forthe in length, vntill it come to the circumference, it will light in the verie poinct A where the circles touche one an other.

¶The, lvij. Theoreme.

If twoo cirtles bee drawen so one without an other, that their edges dooe touche, and a righte line bee drawen from the centre of the one, to the centre of the other, that line shall passe by the place of their touchyng.

¶Example.

The firste circle is A. B. E, and his centre is K. The seconde circle is D. B. C, and his centre is H, the poinct where thei dooe touche is B. Now dooe you see that the line K. H, whiche

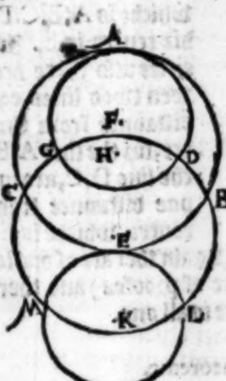


whiche is drawen frō K, that is centre of the firste circle, vnto H, beyng centre of the seconde circle, dooeth passe (as it muste nedes by the poincte B.) whiche is the verye poincte where thei doe touche together.

¶The.lviij.Theoreme.

One circle can not touche an other in more poinctes then one, whether thei touche within, or without.

¶Example.



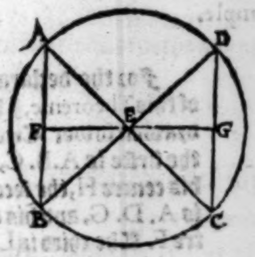
For the declaration of this Theoreme, I haue drawen fower Circles, the firste is A.B. C, and his centre H, the seconde is A. D. G, and his centre F. The third is L.M. and his centre K. The fowerth is D. G. L. M, and his centre E. Nowe as you perceiue the seconde circle A. D. G, toucheth the firste in the inner side, in so muche as it is drawen within the other, and yet it toucheth hym but in one poincte, that is to saie in A, so likewaies the thirde circle L. M, is drawen

without the firste circle, and toucheth hym, as you maie see, but in one place. And now as for the fowerth circle, it is drawen, to declare the diuersitie betwene touching and cutting, or crossing. For one circle maie crosse and cutte a greate many other circles, yet can he not cutte any one in more places then twoo, as the fiue and fiftie Theoreme affirmeth.

¶The.lix.Theoreme.

In euery circle those lines are to bee counted equal, whiche are in like distaunce from the centre. And contrarie waies, thei are in like distaunce from the centre, whiche bee equall.

¶Example.



In this figure you see firste the circle drawen, whiche is A,B,C,D, and his centre is E. In this circle also there are drawen twoo lines equally distaunte from the centre, for the line A,B, and the line D,C, are iuste of one distaunce from the centre, whiche is E, and therefore are thei of one length. Again thei are of one length (as shall bee proued in the booke of proofes) and therefore their distaunce from their centre is all one.

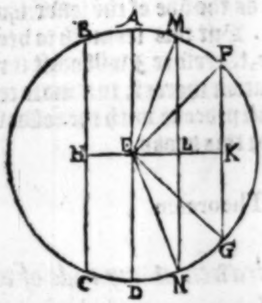
¶The.lx.Theoreme.

In euery circle the longest line is the diameter, and of all the other lines, thei are still longest that be nexte

vnto

vnto the centre, and thei bee the shortest, that bee farthest distaunte from it.

¶Example.



In this circle A.B.C D, I haue drawen firste the diametre, whiche is A.D, whiche passeth (as it must) by the centre E. Then haue I drawen iij. other lines as M.N, whiche is nerer the centre, and F.G, that is farther from the centre. The fowerth line also on the other side of the diametre, that is B.C, is neerer to the centre then the line F.G, for it is like distance as the line M,N. Now saie I, that A.D, beyng the diameter, is the longest of all those lines, and also of any other that maie be drawen within that circle. And the other line M.N. is longer then F.G, because it is nerer to the cētre of the circle then F. G. Also the line F.G, is shorter then the line B.C, for because it is farther from the centre then is the line B.C. And thus maie you iudge of all lines drawen in any circle, howe to knowe the proportion of their lengthe, by the proportion of their distaunce, and contrary waies, how to discerne the proportion of their distaunce by their lengthes, if you knowe the proportion of their lengthe. And to speake of it by the waie, it is a marueilous thyng to consider, that a man maie knowe an exacte proportion betwene two thinges, and yet can not name nor attaine the precise quantitie of those two thynges. As for example, If twoo squares bee sette foorthe, whereof the one containeth in it .v. square feete, and thother containeth fiue and fourtie foote, of like square feete, I am not

not able to tell, no nor yet any manne liuyng, what is the precise measure of the sides, of any of those twoo squares, and yet I can proue by unfallible reason, that their sides bee in a triple proportion, that is to saie, that the side of the greateste square (whiche containeth fowertie and fiue foote) is three tymes so long iuste, as the side of the lesser square, that includeth but fiue foote. But this seemeth to bee spoken out of season in this place, therefore I will omit it now, reseruyng the exacter declaration thereof, to a more conuenient place and tyme, and will procede with the residewe of the Theoremes appoincted for this booke.

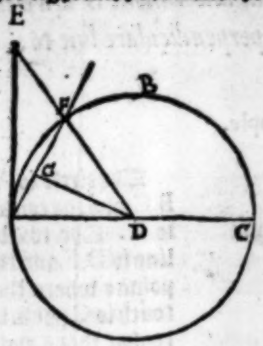
¶The.lxj. Theoreme.

If a righte line bee drawen at any ende of a diametre in perpendiculare fourme, and dooe make a righte angle with the diametre, that righte line shall light without the circle, and yet so ioyntly knitte to it, that it is not poßible to drawe any other right line betwene that saied line, and the circumference of the circle. And the angle that is made in the semicircle is greater then any sharpe angle, that maie bee made of right lines, but the other angle without, is lesser then any that can bee made of right lines.

¶Example.

In this circle A.B.C, the diameter is A.C, the perpendiculare line, whiche maketh a right angle with the diameter is C.A, whiche line falleth without the circle, and yet ioyneth so exactly unto it, that it is not possible to drawe an other

ther right line, betwene the circumference of the circle and it, whiche thyng is so plainly sene of the eye, that it nedeth no farther declaration. For euery manne will easily consent, that betwene the crooked line A.F. (whiche is a parte of the circumference of the circle) and A.E, (whiche is the saied perpendiculare line) there can none other line bee drawen in that place, where thei make the angle. Nowe for the residue of the Theoreme The angle D.A.B. whiche is made in the semicircle, is greater then any sharpe angle, that maie bee made of righte lines, and yet it is a sharpe angle also, in as muche as it is lesser then a right angle, whiche is the angle E.A.D, and the residue of that right angle, which lieth without the circle, that is to saie, E.A.B, is lesser then any sharpe angle that can bee made of right lines also. For as it was before rehearsed, there can no right line bee drawen to the angle betwene the circumference and the right line E.A. Then muste it needes followe, that there can bee made no lesser angle of right lines. And againe, if there can be no lesser then the one, then doeth it sone appere, that there can be no greater then the other, for thei twoo doe make the whole right angle, so that if any corner could be made greater then the other parte, then should the residus bee lesser then the other parte, so that either bothe partes muste bee false, or els bothe graunted to bee true.



¶The.lxij. Theoreme.

If a right line dooe touche a circle, and an other righte line drawen from the centre of the circle, to the

 poincte

poincte where thei touche, that line whiche is drawen from the centre, shall bee a perpendiculare line to the touche line.

¶Example.

The circle is A.B.C, and the centre is F. The touche line is D.E, and the poincte where thei touch is C. Now by reason that a right line is drawē from the centre F. vnto C, whiche is the poincte of the touch therefore saieth Theoreme, that the saied line F. C, muste needes bee a perpendicalare line vnto the touche line D.E.

¶The.lxiij.Theoreme.

If a righte line dooe touche a circle, and an other right line bee drawen from the poincte of their touchyng, so that it dooe make right corners with the touche line, then shall the centre of the circle bee in that same line, so drawen.

¶Example.

The circle is A.B.C, and the centre of it is G. The touch line is D.C.E, and the poincte where it toucheth, is C. Now it appeareth manifeste, that if a right bee drawen from the poincte

poincte where the touch line dooeth ioyne with the circle, and that the saied line do make right corners with the touche line, then must it needes goe by the centre of the circle, and then consequentlie it muste haue the saied centre in hym. For if the saied line should goe beside the centre, as F. C. dooeth, then dooeth it not make right angles with the touche line, whiche in the Theoreme is supposed.

¶The.lxiiij. Theoreme.

If an angle bee made on the centre of a circle, and an other angle made on the circumference of the same circle, and their grounde line be one common portion of the circumference, then is the angle on the centre twise so greate as the other angle on the circumference.

¶Example.

The circle is A.B.C.D, and his centre is E: the angle on the centre is C.E.D. and the angle on the circumference is C. A. D, their common ground line is C. F, D. Nowe saie I that the angle C. E. D, whiche is on the centre, is twise so greate as the angle C. A. D, whiche is on the circumference.

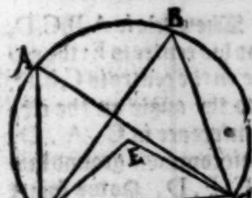
 The

¶The.lv.Theoreme.

Those angles whiche bee made in one cantle of a circle, must needes bee equall together.

¶Example.

Before I declare this Theoreme by an example, it shall bee needefull to declare, what is to bee understande by the woordes of this Theoreme. For the sentence can not bee knowen, unlesse the verie meanyng of the woordes be first understande. Therefore when it speaketh of angles made in one cantle of a circle, it is this to bee understande, that the angle muste touche the circumference: and the lines that doe inclose that angle, muste bee drawen to the extremities of that line, whiche maketh the cantle of the circle. So that if any angle dooe not touche the circumference, or if the lines that inclose that angle, dooe ende in the extremities of the corde line, but ende either in some other parte of the saied corde, or in the circumference, or that any of theim dooe so eande, then is not that angle accoumpted to bee drawen in the saied cantle of the circle. And this promised, nowe will I come to the meanyng of the Theoreme, I sette foorthe a circle, whiche is A. B. C. D, and his Centre E, in this circle I drawe a line C. D, whereby there are made twoo cantels, a more and a lesser. The lesser is D. E. C, and the greater is D. A. B. C. In this greater cantle I drawe twoo angles, the firste is D. A. C: and the seconde is D. B. C, whiche twoo angles by reason thei are made bothe



in

In one cantle of a circle (that is the cantle D.A.B.C.) therefore are thei bothe equall. Now doeth there appere an other triangle, whose angle lighteth on the centre of the circle, and that triangle is D.E.C, whose angle is double to thother angles, as is declared in the fiftie and fower Theoreme, whiche maie stande well enough with this Theoreme, for it is not made in this cantle of the circle, as the other are, by reason that his angle dooeth not lighte in the circumference of the circle, but on the centre it self.

¶The.lxvj.Theoreme.

Euerie figure of fower sides, drawen in a circle, hath his twoo contrarie angles, equall vnto twoo right angles.

¶Example.

The circle is A.B.C.D, and the figure of fower sides in it, is made of the sides B.C, and C.D, and D.A, and A.B. Now if you take any twoo angles that bee contrary, as the angle by A, and the angle by C, I saie that those twoo bee equall to twoo right angles. Also if you take the angle by B, and the angle by D, whiche twoo are also contrarie, those twoo angles are likewaies equall to twoo right angles. But if any manne will take the angle by A, with the angle by B, or D, thei can not bee accoumpted contrarie, no more is not the angle by C, estemed contrary to the angle by B, or yet to the angle by D, for thei onely bee accoumpted contrarie angles, whiche haue no one line common to theim bothe.

Suche is the angle by A, in respecte of the angle by C, for their bothe lines bee distincte, where as the angle by A, and the angle by D, haue one common line A. D, and therefore can not bee accompted contrary angles. So the angle by D, and the angle by C, haue D.C, as a common line, and therefore bee not contrary angles. And this maie you iudge of the residewe, by like reason.

¶The.lxvij.Theoreme.

Vpon one right line there can not bee made twoo cantles of circles, like and vnequall, and drawen toward one parte.

¶Example.

Cantles of circles bee then called like, when the angles that are made in theim bee equall. But nowe for the Theoreme, lette the right line bee A.E.C, on whiche I drawe a cantle of a circle, whiche is A.B.C. Now saith the Theoreme, that it is not possible to drawe an other cantle of a circle, whiche shall bee vnequall vnto this firste cantle, that is to saie, either greater or lesser then it, and yet bee like it also, that is to saie, that the angle in the one, shall be equall to the angle in the other. For as in this example you see a lesser cantle drawen also, that is A.D.C. so if an angle were made in it, that angle would bee greater then the angle made in the cantle A.B.C, and therefore can not thei bee called like cantles, but and if any other cantle were made greater then the firste, then would the angle bee lesser, then that in the

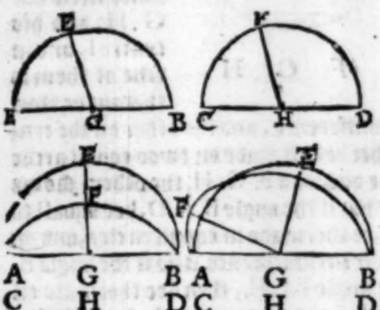
firste, and so neither a lesser, neither a greater cantle can bée made vpon one line with an other, but it will bee vnlike to it also.

¶The.lxviij.Theoreme.

Like cantelles of circles made on equall right lines, are equall together.

¶Example.

What is meante by like cantles you haue heard before, and it is easie to vnderstande, that suche figures are called equall, that bée of one bignesse, so that the one is neither greater, neither lesser then the other. And in this kinde of comparison, thei must so agree, that if the one bee laied on the other, thei shall exactly agree in all their boundes, so that neither shall exceade other.



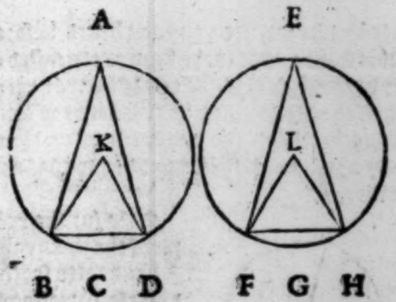
Now for the exāple of the theoreme, I haue sette forthe diuerse varieties of cantles of circles, emongest which the first and second are made vpon equal lines, and are also bothe equall & like. The thirde couple are ioyned in one, and bee neither equall, neither like, but expressyng an absurde deformitie, whiche would followe if this Theoreme were not true. And so in the fowerth couple you maie see, that because thei are not equall cantles, therefore can not thei be like cantles, for necessarily it goeth together, that al cātles of of circles made vpō equall right lines, if thei be like, thei must be equall also.

¶The.lxix.Theoreme.

In

In equall circles, suche angles as bee equall are made vppon equall arche lines of the circumference, whether the angle lighte on the circumference, or on the centre.

¶Example.



First I haue set for an example twoo equal circles, that is A.B.C,D, whose centre is K, and the seconde circle E.F G.H, and his centre L, and in eche of them is ther made twoo angles, one on the circumference, and the other on the centre of eche circle, and thei bee all made on twoo equall arche lines, that is B.C.D, the one, and F. G. H, the other. Nowe saieth the Theoreme, that if the angle B.A.D, bee equall to the angle F.E.H, then are thei made in equall circles, and on equal arche lines of their circumference. Also if the angle B, K.D, bee equall to the angle F.L.H, then bee thei made on the centres of equall circles, and on equall arche lines, so that you must compare those angles together, whiche are made bothe on the centres, or bothe on the circumference, and maie not conferre those angles, whereof one is drawen on the circumference, and the other on the centre. For euermore the angle on the centre in suche sorte, shall bee double to the angle on the circumference, as is declared in the three score and fower Theoreme.

The

¶The.lxx.Theoreme.

In equall circles, those angles whiche bee made on equall arche lines, are euer equall together, whether thei bee made on the centre, or on the circumference.

¶Example.

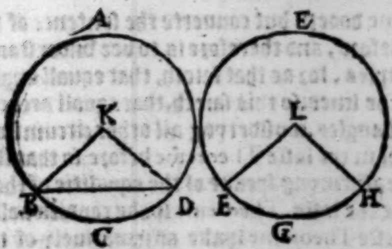
This Theoreme dooeth but converte the sentence of the laste Theoreme before, and therefore is to bee understande by the same examples, for as that saieth, that equall angles occupie equall arche lines: so this saieth, that equall arche lines causeth equall angles, consideryng all other circumstances, as was taught in the laste Theoreme before, so that this Theoreme dooeth affirmyng speake of the equalitie of those angles, of whiche the laste Theoreme spake conditionally. And where the laste Theoreme spake affirmatiuely of the arche lines, this Theoreme speaketh conditionally of them, as thus: If the arche line B.C.D, be equall to the other arche line F.G.H, then is that angle B.A.D, equall to the other angle F.E.H. Or els thus maie you declare it causally: Because the arche line B.C.D, is equall to the other arche line F.G.H, therefore is the angle B.K.D. equall to the angle F.L.H, consideryng that thei are made on the centres of equall circles. And so of the other angles, because those twoo arche lines aforesaied are equall, therefore the angle D.A.B, is equall to the angle F.E.H, for as muche as thei are made on those equall arche lines, and also on the circumference of equall circles. And thus these Theoremes doe one declare an other, and one verifie the other.

¶The.lxxj.Theoreme.

In equall circles, equall right lines beyng drawen, dooe cutte awaie equall arche lines from their circum-

ferences, so that the greater arche line of the one, is equall to the greater arche line of the other, and the lesser to the lesser.

¶Example.



The circle A. B.C.D. is made equall too the circle E.F.G.H and the righte line B.D. is equall too the right line F.H, wherfore it foloweth, that the two arche lines of the circle A.B.D, whiche are cutte from his circumference by the right line B.D, are equall to twoo other arche lines of the circle E.F.H, beyng cutte from his circumferēce, by the right line F.H, that is to saie, that the arche line B.A D, beyng the greater arche line of the firste circle, is equall to the arche line F.E.H, beyng the greater arche line of the other circle. And so in like maner the lesser arche line of the firste circle, beyng B.C.D, is equall to the lesser arche line of the seconde circle, that is F.G.H.

¶The.lxxij.Theoreme.

In equall circles, vnder equall arche lines the right lines that bee drawen are equall together.

¶Example.

This Theoreme is none other, but the conuersion of the laste.

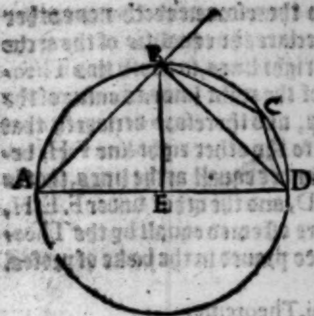
laste Theoreme before, and therefore needeth none other example. For as that did declare the equalitie of the arche lines, by the equalitie of the right lines, so doeth this Theoreme declare the equalnesse of the right lines, to ensue of the equalnesse of the arche lines, and therefore declareth that right line B.D, to bee equall to the other right line F.H, because thei bothe are drawen vnder equall arche lines, that is to saie, the one vnder B.A.D, and the other vnder F.E.H, and those twoo arche lines are estemed equall by the Theoreme laste before, and shall bee proued in the booke of profes.

¶The.lxxiij.Theoreme.

In euery circle, the angle that is made in the halfe circle, is a iuste right angle, and the angle that is made in a cantle greater then the halfe circle, is lesser then a right angle, but that angle that is made in a cantle, lesser then the halfe circle, is greater then a right angle. And moreouer the angle of the greater cantle is greater then a right angle, and the angle of the lesser cantle, is lesser then a right angle.

¶Example.

In this proposition, it shall bee meete to note, that there is a greate diuersitie betwene an angle of a cãtle, and an angle made in a cantle, and also betwene the angle of a semicircle, and the angle made in a semicircle. Also it is meete to note that all angles that be made in the parte of a circle, are made either in a semicircle (whiche is the iuste halfe circle) or els in a cantle of the circle, whiche cantle is either greater or lesser then the semicircle is, as in this figure annexed, you maie perceiue euery one of the thynges seuerally.



Firste the circle is, as you see, A.B.C.D, and his centre E, his diametre is A.D. Then is there a line drawen from A. to B, and so forthe unto F, whiche is without the circle: and an other line also from B, to D, whiche maketh twoo cantles of the whole circle. The greater cantle is D.A.B. and the lesser cantle is B.C.D. In whiche lesser cantle also there are twoo lines that make an angle, the one line is B.C, and the other line is C.D. Now to shewe the difference of the angle in a cantle, and an angle of a cantle: first for an example, I take the greater cantle B.A.D, in whiche is but one angle made, and that is the angle by A, whiche is made of the line A.B, and the line A.D. And this angle is therefore called an angle in a cantle. But nowe the same cantle hath twoo other angles, whiche be called the angles of that cantle, so the twoo angles made of the right line D.B, & the arche line D.A.B, are the twoo angles of this cantle, whereof the one is by D, and the other is by B. Where you must remember, that the angle by D, is made of the right line B, D, and the arche line D.A. And this angle is deuided by an other right line A.E.D, whiche in this case must bee omitted as no line. Also the angle by B, is made of the right line D.B, and of the arche line B.A, and although it bee deuided with twoo other right lines, of whiche the one is the right line B.A, and the other the right line B.E, yet in this case thei are not to bee considered. And by this may you perceiue also, whiche be the angles of the lesser cantle, the first of them is made of the right line B.D, and of the arche line B.C, the secod is made of the right line D,B, and of the arche line D.C. Then are there twoo o-

ther lines, whiche deuide those twoo corners, that is the line B.C, and the line C,D, whiche twoo lines doe meete in the poinct C, and there make an angle, whiche is called an angle made in that lesser cantle, but yet is not any angle of that cantle. And so haue you heard the difference betwene an angle in a cantle, and an angle of a cantle. And in like sort shall you iudge of the angle made in a semicircle, which is distinct from the angles of the semicircle. For in this figure, the angles of the semicircle are those angles, whiche bee by A, and D, and bee made of the right line A.D, beyng the diameter, and of the halfe circumference of the circle, but the angle made in the semicircle, is that angle by B, whiche is made of the right line A.B, and that other right line B.D, whiche as thei meete in the circomference and make an angle, so thei ende with their other extremities at the endes of the diameter. These things promised, now saie I touchyng the Theoreme that euery angle that is made in a semicircle, is a righte angle, and if it bee made in any cantle of a circle, then muste it nedes be either a blunt angle, or els a sharpe angle, and in no wise a righte angle. For if the cantle wherein the angle is made bee greater then the halfe circle, then is that angle a sharpe angle. And generally the greater the cātle is, the lesser is the angle comprised in that cantle: and contrary waies the lesser any cantle is, the greater is the angle that is made in it. Wherefore it must nedes folowe, that the angle made in a cātle lesse then a semicircle, must nedes bee greater then a right angle. So the angle by B, beyng made of a right line A.B, and the right line B.D, is a iuste right angle, because it is made in a semicircle. But the angle made by A, whiche is made of the right line A B, and of the right line A. D, is lesser then a righte angle, and is named a sharpe angle, for as muche as it is made in a cantle of a circle, greater then a semicircle. And contrary waies the angle by C, beeyng made of the righte line B.C, and of the righte line C.D, is greater then a right angle, and is named a blunte angle, because it is made in a cantle of a circle, lesser then a semicircle. But no w

touchyng the other angles of the cantles, I saie accordyng to the Theoreme, that the twoo angles of the greater cantle, whiche are by B. and D, as is before declared, are greater eche of them then a right angle. And the angles of the lesser cantle, whiche are by the same letters B. and D, but bee on the other side of the corde, are lesser eche of them then a right angle, and bee therefore sharpe corners.

¶ The. lxxiiij. Theoreme.

If a righte line dooe touche a circle, and from the poincte where thei touche, a right line be drawen crosse the circle, and deuide it, the angles that the saied line dooeth make with the touche line, are equall to the angles, whiche are made in the cantles of the same circle, on the contrary sides of the line aforesaied.

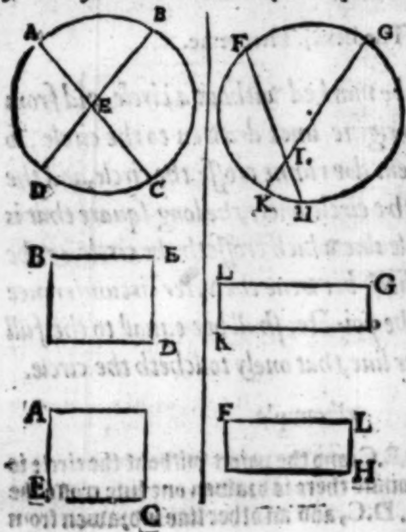
¶ Example.

The circle is A.B.C.D and the touche line is E. F. The poinct of the touchyng is D, from whiche poincte I suppose the line D. B., to be drawen crosse the circle, and to deuide it into twoo cantles, whereof the greater is B.A.D, and the lesser is B.C.D, and in eche of them an angle drawen, for in the greater cantle the angle is by A, and is made of the right lines B. A, and A.D, in the lesser cãtle the angle is by C, and is made of the right lines B. C, and C. D. Now saieth the Theoreme that the angle B.D. F, is equall to the angle, made in the cantle on the other side of the saied

line, that is to saie, in the cantle B.A.D, so that the angle B. D.F, is equall to the angle B.A.D, because the angle B.D.F is on the one side of the line B.D, (whiche is accordyng to the supposition of the Theoreme drawen crosse the circle) and the angle B.A.D, is in the cantle on the other side. Likewaies the angle B.D.E, beyng on the one side of the line B.D, must be equall to the angle B.C.D, (that is the angle by C,) whiche is made in the cantle on the other side of the right line B D. The proofe of al these I doe reserue, as I haue often said, to a conuenient booke, wherein thei shall be all set at large.

¶The.lxxv. Theoreme.

In euery circle when two right lines doe crosse one an other, the likeiamme that is made of the portions of the one line, shall bee equall to the likeiamme made of the partes of the other line. ¶Example.



Because this Theoreme dooeth serue to many vses, and would bee wel vnderstande, I haue set forthe twoo example of it. In the first, the lines by their crossynge dooe make their portiōs somwhat towarde an equalitie. In the seconde, the portions of the lines bee verie farre frō an equalitie, and yet in bothe these and in all other, the

the Theoreme is true. In the first example the circle is A.B. C.D. in whiche the one line A.C. dooeth crosse the other line B.D. in the poincte E. Now if you doe make one liketamme, or longe square of D.B. and E.B, beyng the twoo portions of the line D.E, that longe square shall bee equall to the other longsquare made of A.E. and E.C, beyng the portions of the other line A.C. Likewaies in the seconde example, the circle is F.G.H.K, in whiche the line F.H, dooeth crosse the other line G.K, in the poincte L. Wherefore if you make a liketamme, or longe square of the twoo partes of the line F.H, that is to saie, of F.L, and L.H, that long square will be equal to an other longsquare, made of the twoo partes of the line G.K, whiche partes are G.L, and L.K. These longe squares haue I sette forthe under the circles, containyng their sides, that you maie somewhat whette your owne witte in practisyng this Theoreme, accordyng to the doctrine of the nineteneth conclusion.

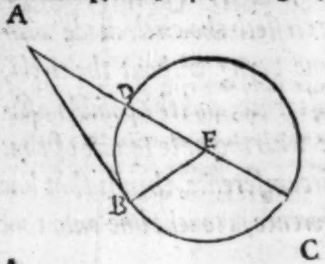
¶ The.lxxvj. Theoreme.

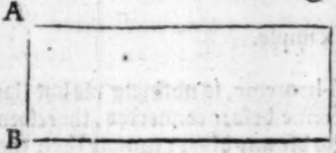
If a poincte be marked without a circle, and from that poincte twoo righte lines drawen to the circle, so that the one of them doe rinne crosse the circle, and the other doe touche the circle onely, the long square that is made of that whole line which crosseth the circle, & the portion of it, that lieth betwene the utter circumference of the circle and the poincte, shall bee equal to the full square of the other line, that onely toucheth the circle.

¶ Example.

This circle is D.B.C, and the point without the circle is A, from whiche poincte there is drawen one line crosse the circle, and that is A, D.C, and an other line is drawen from

the saied pricke, to the marge or edge of the circumference of the circle, and dooeth onely touche it, that is the line A. B. And of that first line A.D.C, you maie perceiue one parte of it, whiche is A. D. to lye without the Circle, betwene the vtter circumference of it, and the point assigned, whiche was A. Now cõcernyng the meanyng of the Theoreme, if you make a longsquare of the whole line A. C, and of that part of it that lieth betwene the circumferẽce and the poincte, whiche is A. D,) that longe square shall bee equall to the full square of the touche line A.B. accordyng not onely as this Figure sheweth, but also the saied nineteneth Conclusion dooeth proue, if you liste to examine the one by the other.





¶The.lxxvij. Theoreme.

If a poincte bee aßigned without a circle, and from that poincte twoo right lines bee drawen to the circle, so that the one dooe crosse the circle, and the o-

ther doe ende at the circumference, and that the longe square of the line, whiche crosseth the circle made with the portion of the same line beyng without the circle, betwene the vtter circumference, and the poincte assigned, dooe equally agree with the iuste square of that line that endeth at the circumference, then is that line so endyng on the circumference, a touche line vnto that circle.

¶Example.

In as muche as this Theoreme, is nothyng els but the sentence of the laste Theoreme before conuerted, therefore it shall not bee needefull, to vse any other example then the same, for as in that other Theoreme, because the one line is a touche line, therefore it maketh a square iuste equall, with the longsquare made of that whole line, whiche crosseth the circle, and his portion liyng without the same circle. So saith this Theoreme: that if the iuste square of the line, that endeth on the circumference, bee equall to that longsquare, whiche is made as for his longer sides of the whole line, whiche commeth from the poincte assigned, and crosseth the circle, and for his other shorter sides, is made of the portion of the same line, liyng betwene the circumference of the circle, and the poincte assigned, then is that line whiche endeth on the circumference a right touche line, that is to saie, if the full square of the right line A.B, bee equall to the long square, made of the whole line A. C. as one of his lines, and of his portion A.D, as his other line, then must it nedes bee, that the line A.B. is a right touch line vnto the circle D.B.C And thus for this tyme, I make an ende of the Theoremes.

FINIS.

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Anno Domini.
1574.

Richard Wyld

Amen

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amen

Lee

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farrington

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